4. [12 points] Another farmer notices the plague of grasshoppers has spread to his crop. He also visits the pest control company and requests a cheaper pesticide. This new pesticide is capable of eliminating the grasshoppers at a rate that decreases with time. Specifically, the rate at which grasshoppers are killed is given by the function $f(t) = \frac{3}{10}(4 - t)$ in thousands of grasshoppers per week at t weeks after the pesticide application. There is no pesticide remaining after 4 weeks. Suppose there are 3000 grasshoppers at the time the pesticide is applied.

Let $Q(t)$ the population of grasshoppers (in thousands) t weeks after this cheaper pesticide is applied to the crop. Then for $0 \leq t \leq 4$, $Q(t)$ satisfies

$$\frac{dQ}{dt} = \frac{Q}{5} - f(t).$$

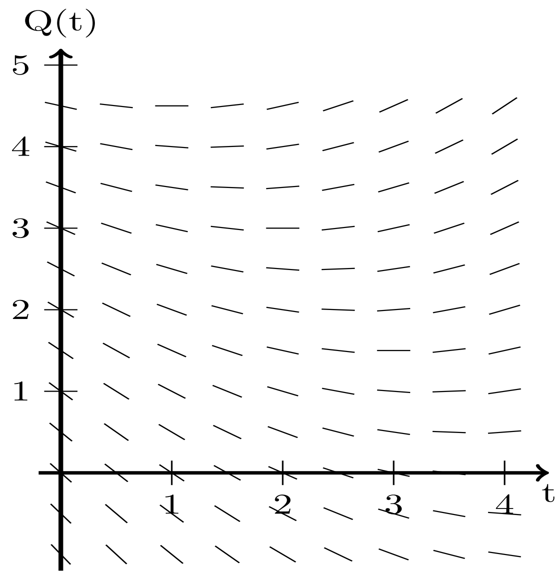
a. [1 point] Is this differential equation separable?

b. [7 points] Using Euler's method, fill the table with the amount of grasshoppers (in thousands) in the crop during the first week. Show all your computations.

t	0	$\frac{1}{2}$	1
$Q(t)$			

(problem 4 continued)

Use the slope field of the differential equation satisfied by $Q(t)$ to answer the following questions.



c. [2 points] Does this equation have any equilibrium solutions in the region shown? List each equilibrium solution and determine whether it is stable or unstable. **Justify your answer**.

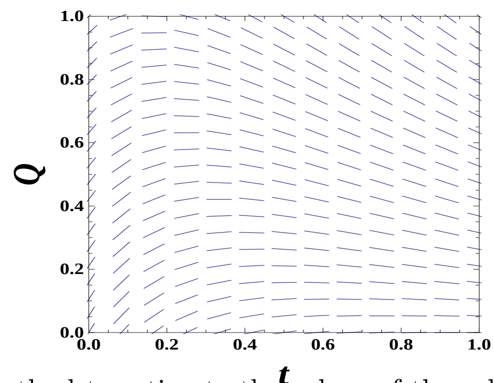
d. [2 points] If the farmer's goal is to kill all the grasshoppers in his crop, will the pesticide be effective in this case? Draw the solution $Q(t)$ on the slope field.

4. [11 points] A restaurant installs a kitchen ventilation system to control the amount of grease in the air due to cooking. The ventilation system reduces the amount of grease in the air by 90 percent every hour. Let $Q(t)$ be the amount in grams of grease in the air t hours after the ventilation is activated. Then Q satisfies the differential equation

$$\frac{dQ}{dt} = 2e^{-5t} - \frac{9}{10}Q,$$

where $2e^{-5t}$ is the rate at which the kitchen produces grease in grams per hour at time t.

a. [2 points] The slope field of the differential equation is shown below. Suppose that the air in the kitchen initially has 0.4 grams of grease. Sketch the solution curve in the slope field.



b. [7 points] Use Euler's method to estimate the values of the solution curve $Q(t)$ through $(0, 0.4)$ for all values of t given in the table below. Show all your work.

t	0	$\frac{1}{3}$	$\frac{2}{3}$	1
$Q(t)$				

c. [2 points] Does your approximation for $Q(1)$ using Euler's method give an overestimate or an underestimate? Justify.

2. [8 points] Wild rabbits were introduced to Australia in 1859. The behavior of the rabbit population P in Australia at a time t years after 1859 was modeled by the differential equation

$$P' = P + e^{-t}.$$

a. [4 points] For what value of B is

$$P = 3e^{t} + Be^{-t}$$

a solution to the differential equation? Be sure to show clearly how you got your answer.

b. [4 points] Suppose that the rabbit population in 1859 was 24 rabbits. Historians used Euler's method with $\Delta t = \frac{1}{2}$ to estimate the rabbit population in 1861. Is their answer an overestimate or and underestimate? Give a brief justification of your answer.

Overestimate **Underestimate**

3. [6 points] Write an explicit expression involving integrals which gives the arc length of **one petal** of the polar rose $r = 3\cos(5\theta)$. Your answer should not contain the letter 'r'. Do not evaluate any integrals.