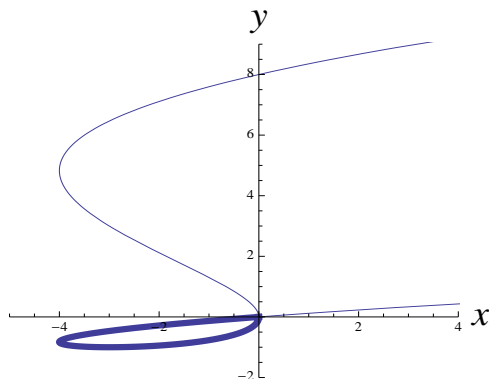


2. [12 points] Consider a particle whose trajectory in the  $xy$ -plane is given by the parametric curve defined by the equations

$$x(t) = t^4 - 4t^2, \quad y(t) = t^2 - 2t,$$

for  $-3 \leq t \leq 3$ . Show all your work to receive full credit.

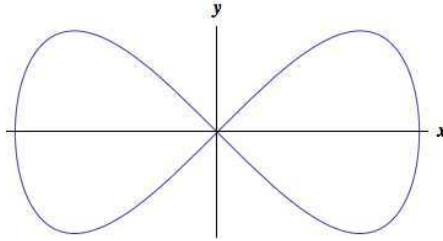
- a. [3 points] Is there any value of  $t$  at which the particle ever comes to a stop? Justify.
- b. [2 points] For what values of  $t$  does the path of the particle have a vertical tangent line?
- c. [3 points] What is the lowest point  $(x, y)$  on the curve?
- d. [2 points] At what values of  $t$  does the particle pass through the origin?
- e. [2 points] The graph of the curve traced by these parametric equations is shown below. Find an expression for the length of the closed loop marked in the graph.



6. [13 points] A particle moves along the path given by the parametric equations

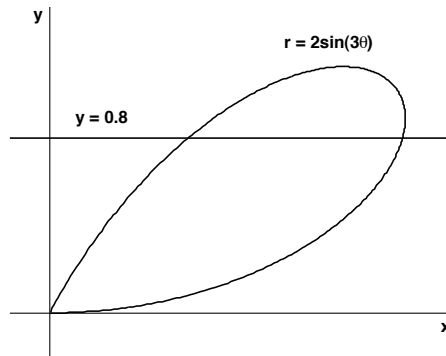
$$x(t) = a \cos t \quad y(t) = \sin 2t \quad \text{for } 0 \leq t \leq 2\pi.$$

where  $a$  is a positive constant. The graph of the particle's path in the  $x$ - $y$  plane is shown below. In the questions below, show all your work to receive full credit.



- a. [2 points] At which values of  $0 \leq t \leq 2\pi$ , does the particle pass through the origin?
- b. [5 points] For what values of  $a$  are the two tangent lines to the curve at the origin perpendicular? Hint: Two lines are perpendicular if the product of their slopes is equal to  $-1$ .
- c. [4 points] At what values of  $0 \leq t \leq 2\pi$ , does the curve have horizontal tangents?
- d. [2 points] Find an expression that computes the length of the curve.

8. [14 points] Consider the area contained above the line  $y = 0.8$ , and below the curve  $r = 2\sin(3\theta)$ . You may find the following figure helpful.



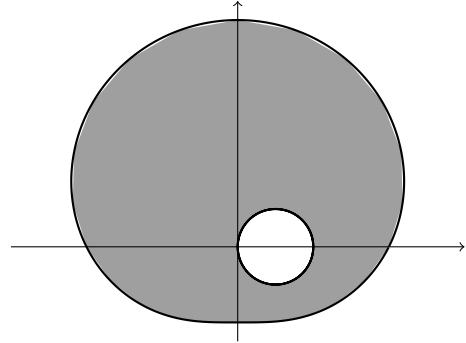
- a. [4 points] Find the  $(x, y)$  coordinates for the two points where  $y = 0.8$  and  $r = 2\sin(3\theta)$  intersect as shown in the figure above. Show enough work to support your answer.
- b. [4 points] Write an expression for the area that is specified. You do not need to evaluate your expression.
- c. [6 points] Calculate the perimeter that surrounds the specified area. You may round your final answer to two decimal places.

5. [11 points] Franklin's robot army is surrounding you!  
 a. [6 points] Consider the polar curves

$$r = \cos(\theta)$$

$$r = \sin(\theta) + 2$$

Franklin's robot army occupies the shaded region between these two curves. Write an expression involving integrals that gives the **area** occupied by Franklin's robot army. Do not evaluate any integrals.



- b. [5 points] Your friend, Kazilla, pours her magic potion on the ground. Suddenly, a flock of wild chickens surrounds you. The chickens occupy the shaded region enclosed within the polar curve  $r = 1 + 2\cos(\theta)$  as shown below. Write an expression involving integrals that gives the **perimeter** of the region occupied by the flock of wild chickens. Do not evaluate any integrals.

