Math 116 — Practice for Exam 1

Generated January 9, 2017

Name: SOLUTIONS

Instructor: ______________________ Section Number: __________

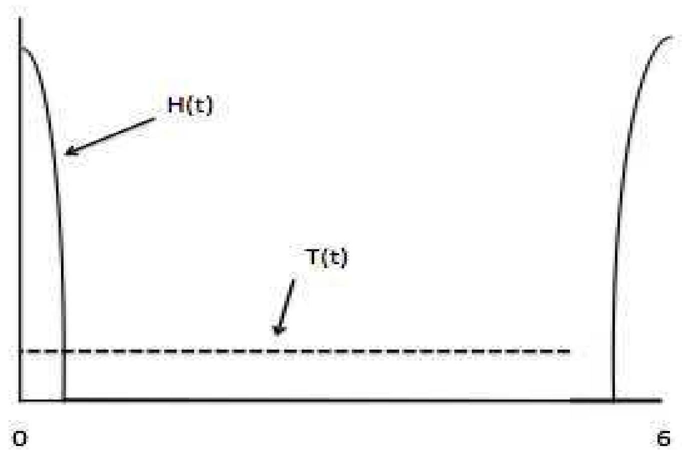
1. This exam has 5 questions. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
2. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you hand in the exam.
3. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
4. Show an appropriate amount of work (including appropriate explanation) for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
5. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
6. If you use graphs or tables to obtain an answer, be certain to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
7. You must use the methods learned in this course to solve all problems.

Semester	Exam	Problem	Name	Points	Score
Winter 2013	1	8	tortoise+hare	11	
Fall 2013	1	7	caffeine	6	
Winter 2014	1	4		9	
Fall 2014	1	2		15	
Fall 2016	1	2	sparrows	16	
Total				57	

Recommended time (based on points): 51 minutes

8. [11 points] A tortoise and a hare decide to race. They decide to race a straight 5 kilometer course. The race starts at 12pm. The hare is much faster than the tortoise, so he's confident that he'll win. The hare runs very fast for 30 minutes, getting to what it knows is the half-way point. The hare is tired (it had been studying for exams the night before), so it decides to take a nap. It falls asleep for 5 hours, wakes up, discovers that (now that it's 5:30) it's dark, and runs to the finish line, arriving at 6pm. When it gets there, it's surprised to see the tortoise is already there. "I hope you enjoyed your nap! I've been here for an hour, since 5 o'clock!" the tortoise says. "Steady and slow is the way to go: I kept going the same speed the whole time."

Let $H(t)$ be the hare's velocity and $T(t)$ be the tortoise's velocity, in km per hour, where t is measured in hours after 12pm.



Let

$$R(t) = \int_0^t H(s)ds - \int_0^t T(s)ds.$$

a. [1 point] At times when $R(t) > 0$, who is winning the race?

Solution: The hare

b. [2 points] What is the practical interpretation of the function $|R(t)|$? Include units.

Solution: $|R(t)|$ is the distance in km between the tortoise and the hare t hours after 12pm.

c. [3 points] For what values of $0 \le t \le 6$, does $R(t) = 0$?

Solution: $t = 0, t = 2.5, t = 6$.

d. [2 points] For what values of $0 \le t \le 6$ is the function $\frac{dR}{dt} < 0$?

Solution: $0.5 < t < 5$.

e. [3 points] Write down a definite integral that represents the hare's average velocity from 12 to 12:30. What is the value of the hare's average velocity during this time?

Solution: $\frac{1}{.5}\int_0^{1/2} H(s)ds$. We know that $\int_0^{1/2} H(s)ds = 2.5$, because the Hare has gotten halfway by 12:30. Therefore, the average velocity is 5 km/hr.

7. [6 points] Let $g(t)$ be the concentration of caffeine (in milligrams per liter) in the bloodstream of a Math 116 GSI, t hours after calculus exam grading begins. Define

$$G(t) = \int_0^t g(x)dx,$$

and let $A(t) = \frac{1}{t}\, G(t)$.

a. [3 points] What is the practical interpretation of the statement $A(4) = 70$? Include units.

Solution: The average concentration of caffeine in the bloodstream of a Math 116 GSI, during the first 4 hours of calculus exam grading, is 70 milligrams per liter.

b. [3 points] Find $A'(t)$.

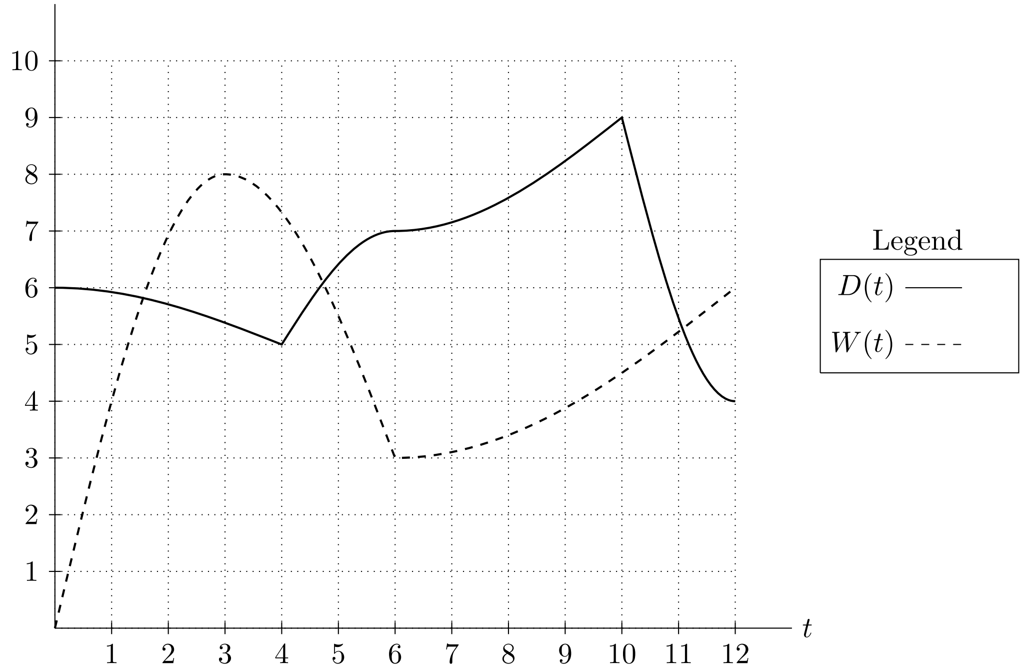
Solution: By the product rule and the fundamental theorem of calculus,

$$A'(t) = \frac{d}{dt}\left(\frac{1}{t}G(t)\right) = -\frac{1}{t^2}\int_0^t g(x)dx + \frac{1}{t}g(t).$$

Some alternate (equivalent) answers:

$$A'(t) = -\frac{1}{t^2}G(t) + \frac{1}{t}g(t), \qquad A'(t) = -\frac{1}{t}A(t) + \frac{1}{t}g(t).$$

4. [9 points] A Swiss bank is constantly receiving deposits and withdrawals of money. Let $D(t)$ be the deposit rate (the rate at which money is going into the bank) and $W(t)$ be the withdrawal rate (the rate at which money is being taken out of the bank), both in millions of dollars/month, where t is measured in months since January 1st 2013. Suppose that on January 1st 2013 the bank has $50 million. A graph of the two functions is shown below.



a. [4 points] Write an expression that gives the amount of money in the bank at time t. Include units.

Solution: $M(t) = \int_0^t (D(x) - W(x))dx + 50$ million dollars.

Alternatively $M(t) = 10^6 \int_0^t (D(x) - W(x)dx) + 5 * 10^7$ dollars

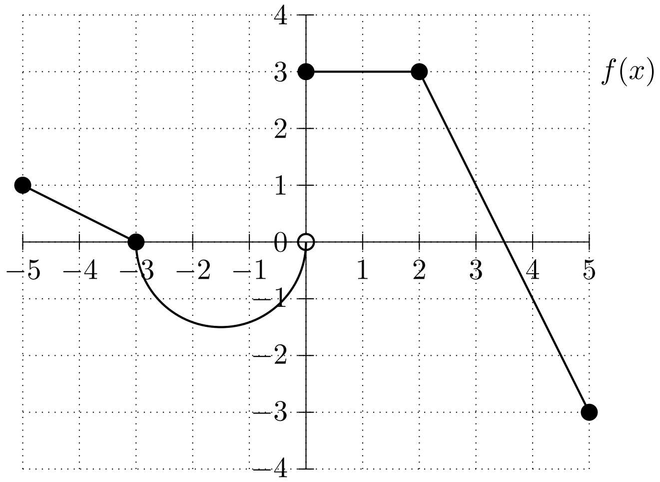
b. [3 points] Write an expression that gives the average rate of change of the amount of money in the bank, in millions of dollars per month, during the year 2013.

Solution: $\frac{1}{12}\int_0^{12}(D(t) - W(t))dt.$

c. [2 points] Estimate the date in 2013 when the bank has the most money in it. You do not need to show your work.

Solution: $t \approx 11$ or approximately December 1st 2013.

2. [15 points] Below is a graph of the function $f(x)$, comprised of line segments and a semicircle. Let $F(x)$ be an anti-derivative of $f(x)$ with $F(2) = 3$.



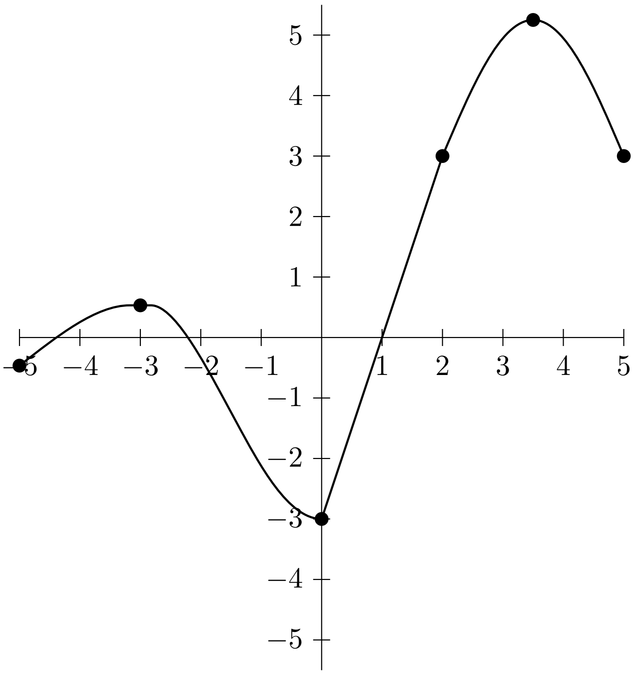
a. [4 points] Find both coordinates of the points where $F(x)$ attains its maximum and minimum values on the interval $-5 \le x \le 5$. No explanation is necessary.

Min:(0 , −3) **Max:** (3.5 , 5.25)

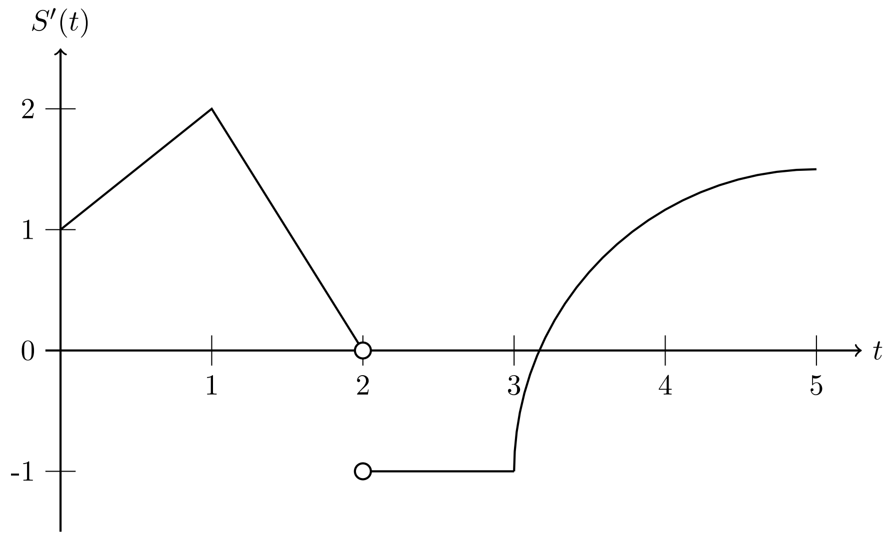
b. [4 points] Find all values of x where $F(x)$ is concave down. Write your answer in the space provided. No explanation is necessary.

$-5 < x < -1.5$ **and** $2 < x < 5$

c. [7 points] Carefully sketch a graph of $F(x)$ on the axes provided below. Be sure to clearly indicate continuity and differentiability in your graph.



2. [16 points] The local sparrow population has been fluctuating unnaturally, and Raymond Green has five months of data to prove it. Let $S(t)$ denote the local sparrow population **in thousands**, t months after Green started collecting data. A graph of $S'(t)$, the rate of population growth, is below. Assume there are 2000 sparrows at $t = 1$.



a. [1 point] At which t-value(s) is the sparrow population increasing the fastest?

Solution: The population is increasing fastest at $t = 1$.

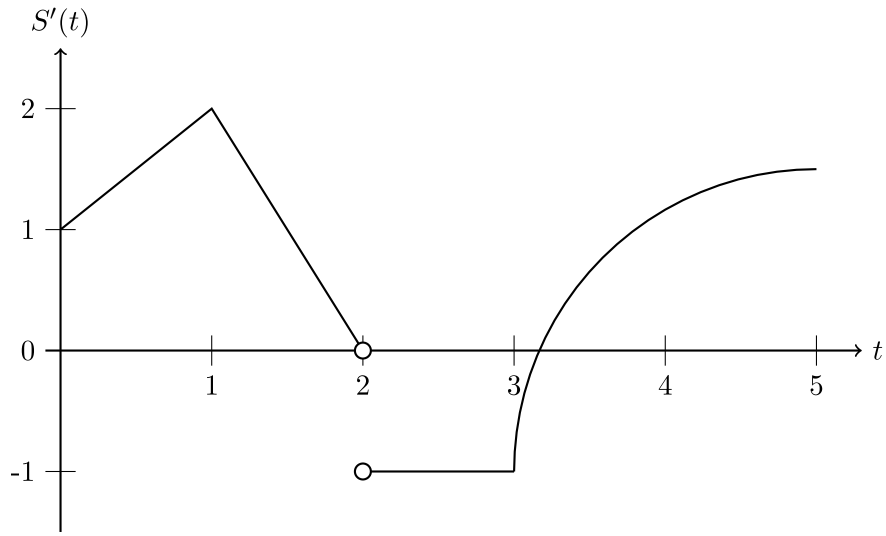
b. [3 points] What is the local sparrow population at $t = 0$, $t = 2$ and $t = 3$?

Solution: The population is 500 at $t = 0$, 3000 at $t = 2$, and 2000 at $t = 3$.

c. [2 points] At which t-values is the population at its highest and lowest?

Solution: The population is highest at $t = 5$ and lowest at $t = 0$.

2 (continued). Recall that $S(t)$ is the local sparrow population **in thousands**, t months after Green began collecting data.



d. [10 points] Sketch a graph of $S(t)$ on the axes below, recalling that there are 2000 sparrows at $t = 1$. Label your vertical axis. Make sure that concavity and local extrema are clear.

Solution: