5. [12 points] Suppose a curve in the plane is given by the equation

$$\sin(\pi xy) = y - 1.$$

a. [3 points] Verify that the point $(x, y) = (1, 1)$ is on the curve.

b. [5 points] Calculate $\dfrac{dy}{dx}$.

c. [4 points] Find the equation for the tangent line to the curve at the point $(1, 1)$.

3. [12 points] The following questions relate to the implicit function

$$y^2 + 4x = 4xy^2.$$

a. [4 points] Compute $\dfrac{dy}{dx}$.

b. [4 points] Find the equation for the tangent line to this curve at the point $(\frac{1}{3}, 2)$.

c. [4 points] Find the x- and y-coordinates of all points at which the tangent line to this curve is vertical.

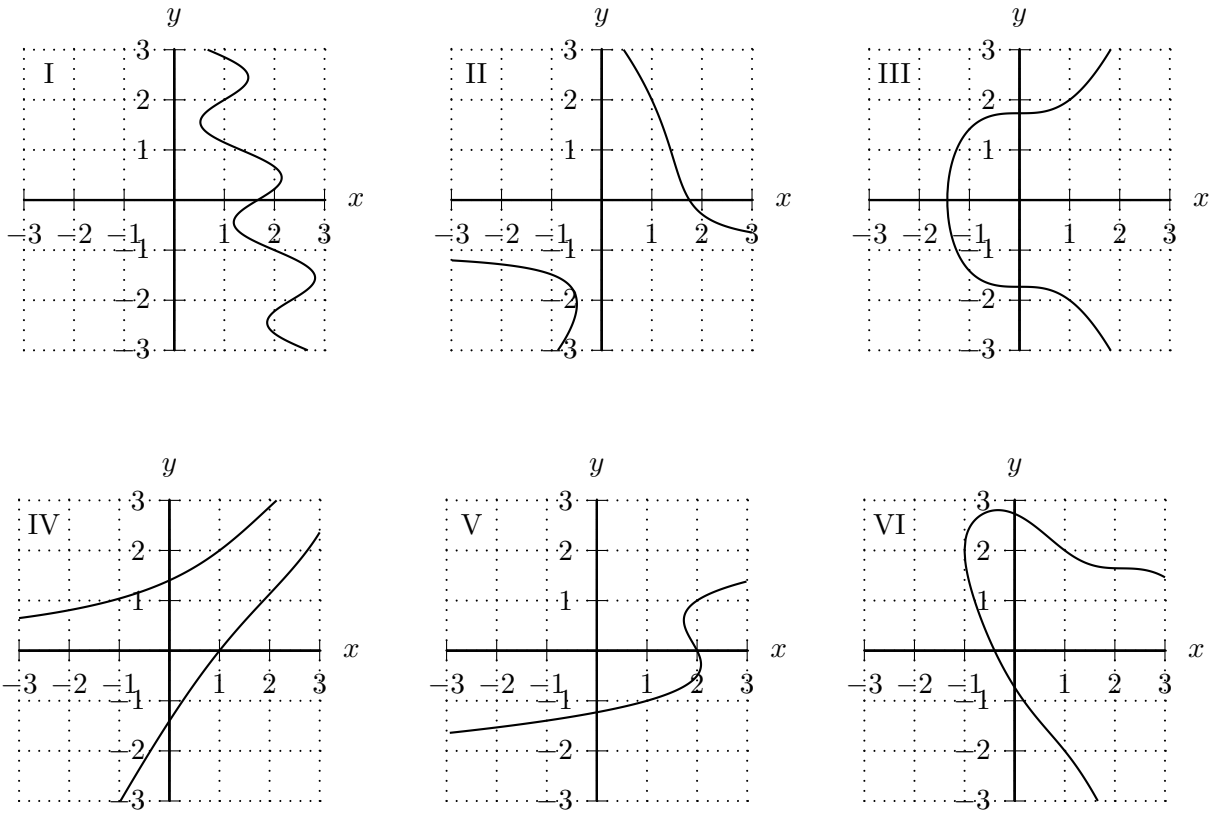
11. [5 points] A curve $\mathcal{C}$ gives y as an implicit function of x. The curve $\mathcal{C}$ passes through the point $(1, 2)$ and satisfies

$$\frac{dy}{dx} = \frac{y^2 - 2xy + 4y - 5}{4(y - x)}.$$

a. [1 point] One of the values below is the slope of the curve $\mathcal{C}$ at the point $(1, 2)$. Circle that one value.

Answer: The slope at $(1, 2)$ is $\quad \frac{1}{4} \quad \frac{1}{3} \quad \frac{1}{2} \quad \frac{5}{8} \quad \frac{2}{3} \quad \frac{3}{4} \quad \frac{4}{5}$

b. [4 points] One of the following graphs is the graph of the curve $\mathcal{C}$.
Which of the graphs I-VI is it? To receive any credit on this question, you must circle your answer next to the word "Answer" below.



Remember: To receive any credit on this question, you must circle your answer next to the word "Answer" below.

Answer: I II III IV V VI

7. [6 points] A curve $\mathcal{C}$ gives y as an implicit function of x. This curve passes through the point $(-2, 1)$ and satisfies

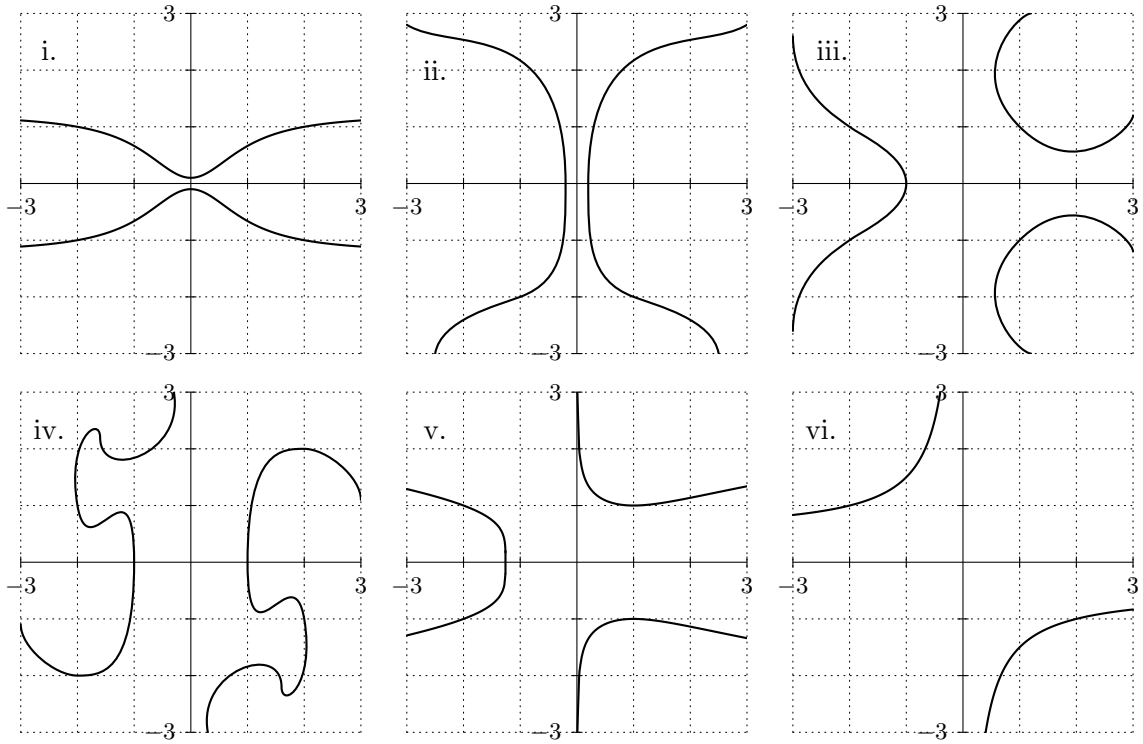
$$\frac{dy}{dx} = \frac{x^2 - y^4}{2xy^3}.$$

a. [1 point] One of the values below is the slope of the curve $\mathcal{C}$ at the point $(-2, 1)$. Circle that one value.

Answer: The slope at $(-2, 1)$ is

$$-\frac{3}{16} \qquad -\frac{1}{4} \qquad -\frac{3}{8} \qquad -\frac{1}{2} \qquad -\frac{5}{8} \qquad -\frac{3}{4} \qquad -\frac{15}{16}$$

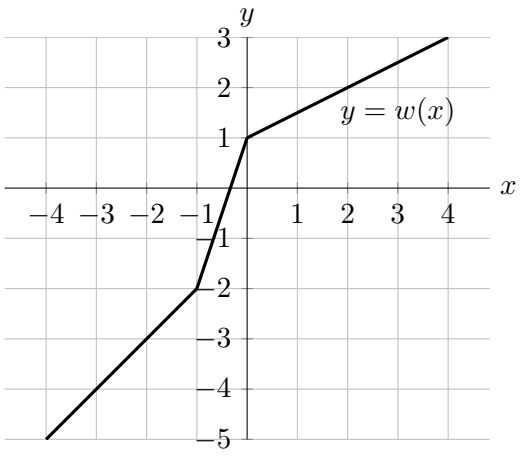
b. [5 points] One of the following graphs is the graph of the curve $\mathcal{C}$. Which of the graphs i-vi is it? To receive any credit on this question, you must circle your answer next to the word "Answer" below.



Remember: To receive any credit on this question, you must circle your answer next to the word "Answer" below.

Answer: i. ii. iii. iv. v. vi.

4. [10 points] A portion of the graph of the function $w(x)$ is shown below.



For each of the parts below, find the value of the given quantity. If there is not enough information provided to find the value, write NOT ENOUGH INFO. If the value does not exist, write DOES NOT EXIST. You are not required to show your work on this problem. However, limited partial credit may be awarded based on work shown. All your answers must be in **exact** form.

a. [2 points] Let $k(x) = w^{-1}(x)$. Find $k'(-1.5)$.

Answer: $k'(-1.5) =$ _____

b. [2 points] Let $h(u) = \ln(3w(u))$. Find $h'(1)$.

Answer: $h'(1) =$ _____

c. [2 points] Let $n(x) = \dfrac{w(x)}{1-x^2}$. Find $n'(-2)$.

Answer: $n'(-2) =$ _____

d. [2 points] Let $s(x)$ be the exponential function $s(x) = 4^{w(x)}$. Find $s'(2)$.

Answer: $s'(2) =$ _____

e. [2 points] Let $p(x) = x \cdot w^{-1}(x)$. Find $p'(-1)$.

Answer: $p'(-1) =$ _____

10. [4 points] Let a and b be constants. Consider the curve $\mathcal{C}$ defined by the equation

$$\cos(ax) + by\ln(x) = 3 + y^3.$$

Find a formula for $\frac{dy}{dx}$ in terms of x and y. The constants a and b may appear in your answer. To earn credit for this problem, you must compute this by hand and show every step of your work clearly.

Answer: $\dfrac{dy}{dx} =$

11. [6 points] Let $h(x) = x^x$. For this problem, it may be helpful to know the following formulas:

$$h'(x) = x^x\left(\ln(x)+1\right) \qquad \text{and} \qquad h''(x) = x^x\left(\frac{1}{x} + (\ln(x)+1)^2\right).$$

a. [2 points] Write a formula for $p(x)$, the local linearization of $h(x)$ near $x = 1$.

Answer: $p(x) =$ _____

b. [4 points] Write a formula for $u(x)$, the quadratic approximation of $h(x)$ at $x = 1$. (Recall that a formula for the quadratic approximation $Q(x)$ of a function $f(x)$ at $x = a$ is $Q(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2$.)

Answer: $u(x) =$ _____

4. [13 points] Let $f(x) = e^{\sin\sqrt{x}}$. Let P be the point on the graph of f at which $x = 4\pi^2 (\approx 39.4784)$.

a. [3 points] Calculate $f'(x)$.

b. [4 points] Find an **exact** formula for the tangent line $L(x)$ to $f(x)$ at P. **Exact** means your answer should not involve any decimal approximations.

c. [2 points] Use your formula for $L(x)$ to approximate $e^{\sin\sqrt{38}}$.

d. [4 points] Recall that the error, $E(x)$, is the actual value of the function minus the value approximated by the tangent line. Given the fact that in this case $E(39) \approx 0.000613$ and $E(40) \approx 0.000719$, would you expect $f''(4\pi^2)$ to be positive or negative? Explain, without doing any calculations.

6. [9 points] A group of biology students is studying the length L of a newborn corn snake (in cm) as a function of its weight w (in grams). That is, $L = G(w)$. A table of values of $G(w)$ is shown below.

w	5	10	15	20	25
$G(w)$	24.5	31.6	38.7	44.7	50
$G'(w)$	2.23	1.58	1.30	1.12	1.05

Assume that $G'(w)$ is a differentiable and decreasing function for $0 < w < 25$.

a. [2 points] Find a formula for $H(w)$, the tangent line approximation of $G(w)$ near $w = 20$.

Answer: $H(w) =$ _____

b. [1 point] Use the tangent line approximation of $G(w)$ near $w = 20$ to approximate the length of a corn snake that weighs 22 grams.

Answer: _____

c. [2 points] Is your answer in part (b) an overestimate or an underestimate? Circle your answer and write a sentence to justify it.

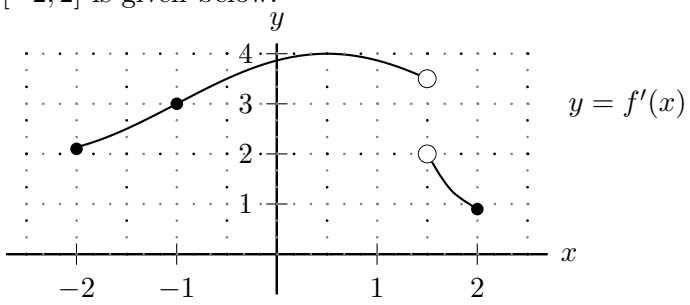
Circle one: Overestimate Underestimate CANNOT BE DETERMINED

Justification:

d. [4 points] In their study of the growth of corn snakes, they found the results of a recent article that states that the average weight w of a corn snake (in grams) t weeks after being born is given by $w = \frac{1}{5}t^2$. Let $S(t) = G\left(\frac{1}{5}t^2\right)$ be the length of a corn snake t weeks after being born. Find a formula for $P(t)$, the tangent line approximation of $S(t)$ near $t = 5$.

Answer: $P(t) =$ _____

3. [8 points] Suppose $f(x)$ is a function that is continuous on the interval $[-2, 2]$. The graph of $f'(x)$ on the interval $[-2, 2]$ is given below.



a. [3 points] Let $L(x)$ be the local linearization of $f(x)$ at $x = -1$. Using the fact that $f(-1) = 4$, write a formula for $L(x)$.

Answer: $L(x) =$ _____

b. [2 points] Use your formula for $L(x)$ to approximate $f(-0.5)$.

Answer: $f(-0.5) \approx$ _____

c. [3 points] Is your answer from part (b) an overestimate or an underestimate of the actual value of $f(-0.5)$? Justify your answer.

Circle one: overestimate underestimate CANNOT BE DETERMINED

Justification: