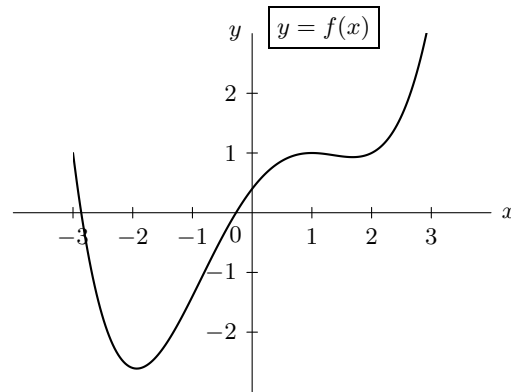
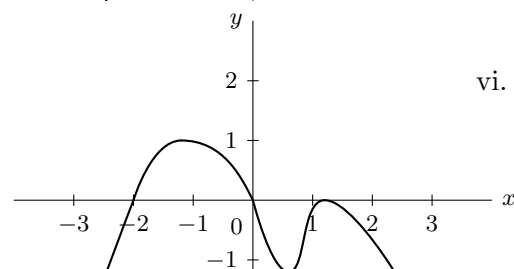
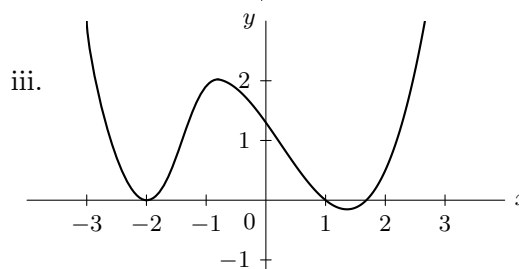
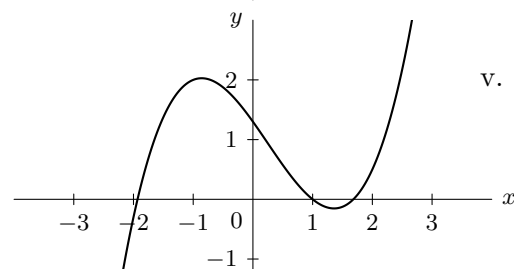
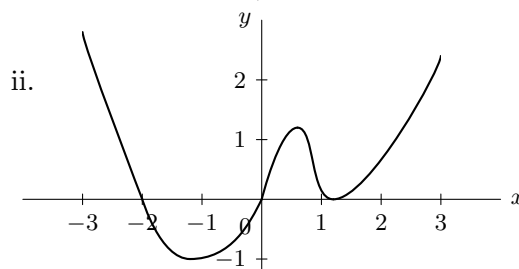
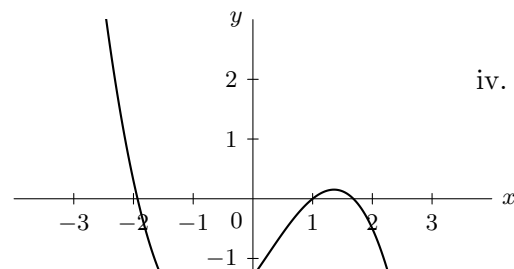
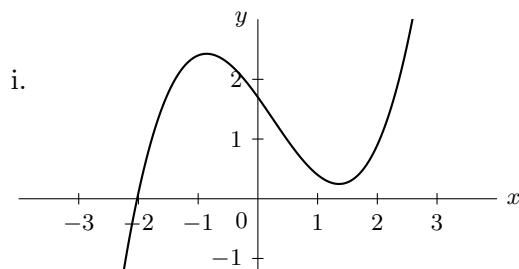


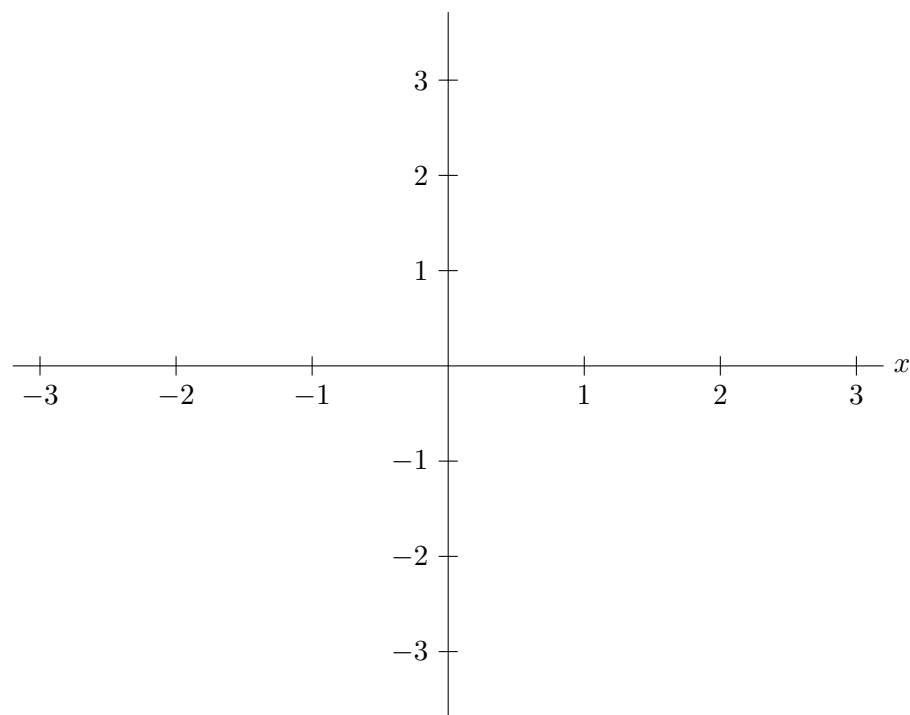
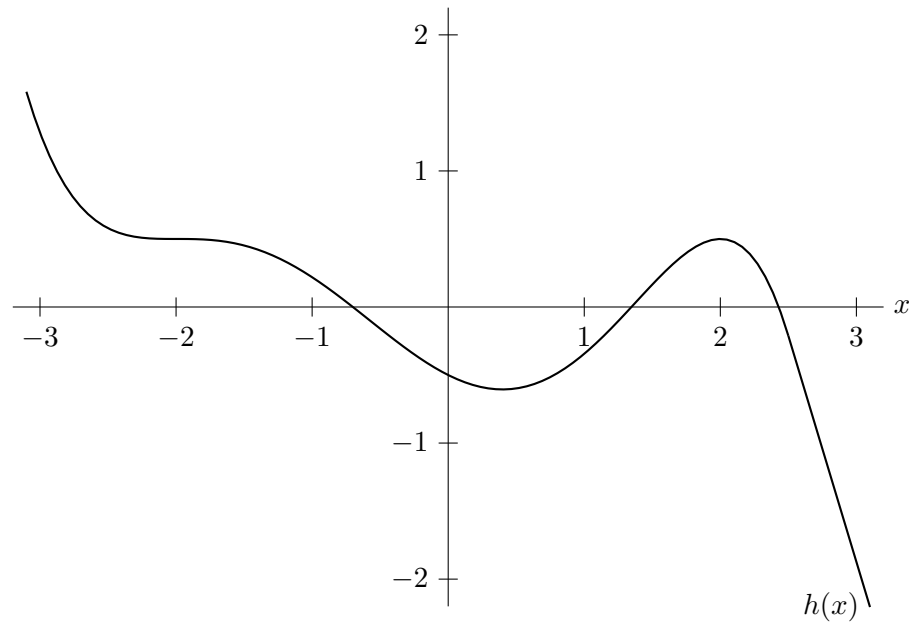
1. [5 points] Below is the graph of a function $f(x)$.



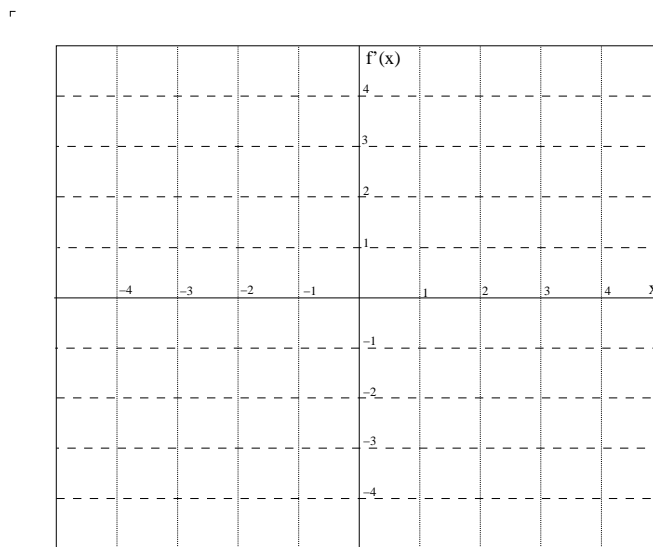
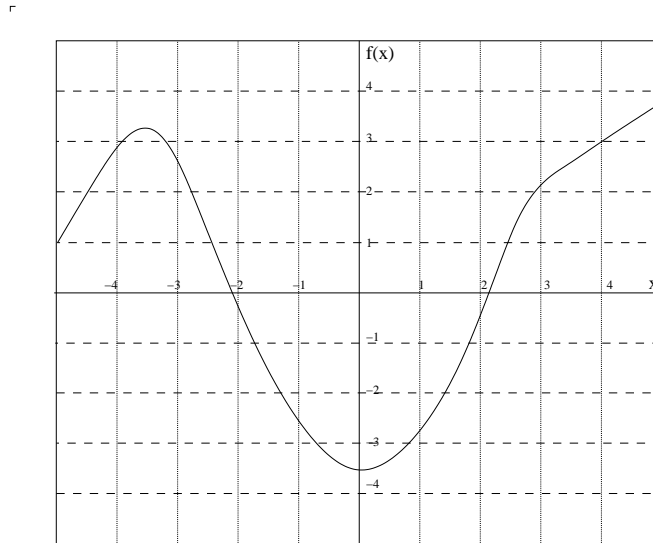
There are six graphs shown below. Circle the one graph that could be the graph of the derivative $f'(x)$.



9. [10 points] Given below is the graph of a differentiable function $h(x)$ which is linear for $x > 2.5$. On the second set of axes, sketch a possible graph of $h'(x)$. Be sure your graph is drawn carefully.



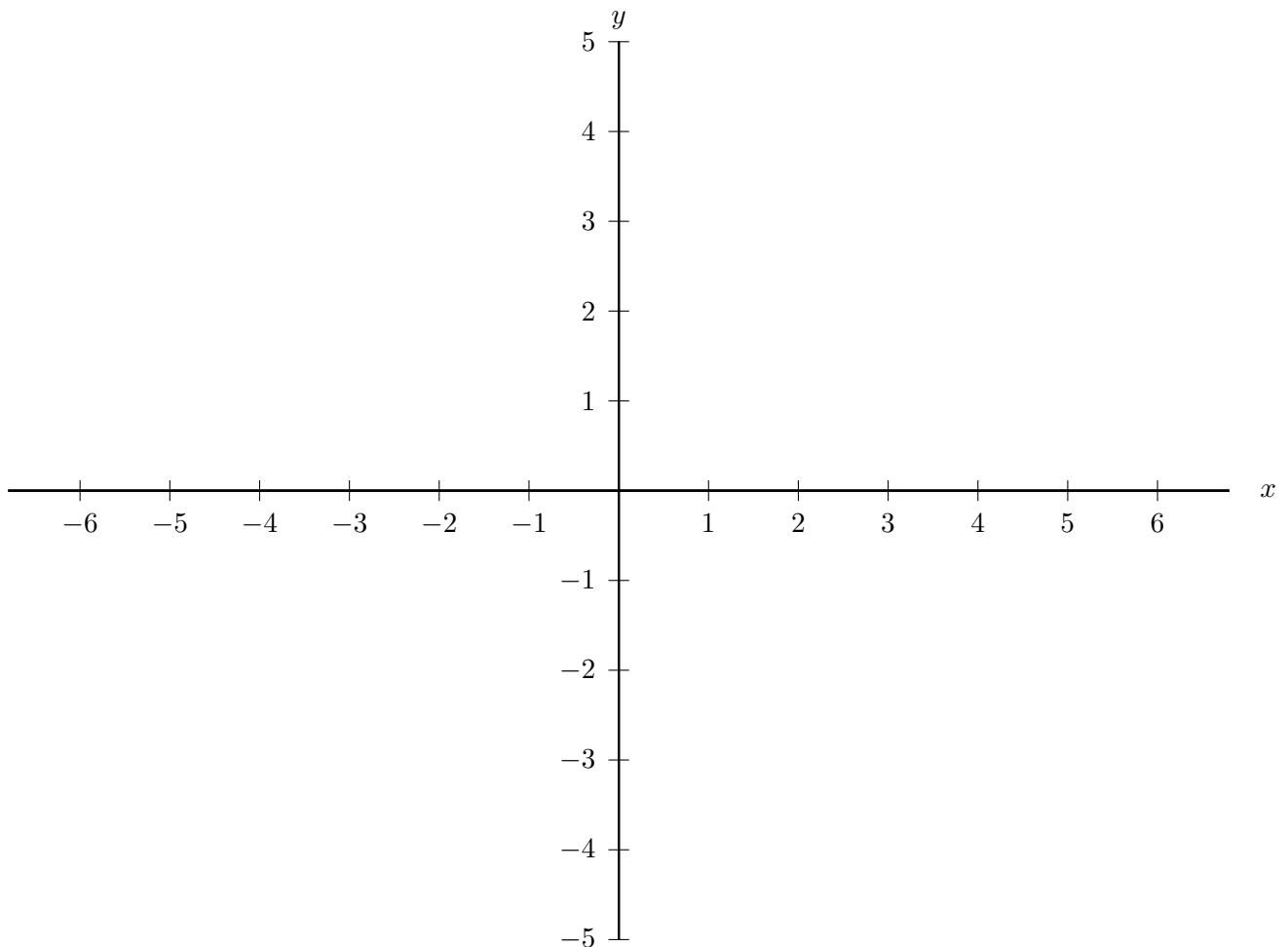
- (8.) (10 points) The graph of $y = f(x)$ is given in the figure below. On the second set of axes below, sketch the graph of the derivative of f . Use the scale on the graph of f to help estimate values for f' .



10. [10 points] On the axes provided below, sketch the graph of a single function $y = h(x)$ satisfying all of the following:

- $h(x)$ is defined for all x in the interval $-6 < x < 6$.
- $h'(x) < 0$ for all $x < -3$.
- $\lim_{x \rightarrow -2^+} h(x) = -1$.
- $h'(0) = 0$.
- The average rate of change of $h(x)$ between $x = -1$ and $x = 2$ is 1.
- $h(x)$ is not continuous at $x = 3$.
- $h(x) > 0$ for all $x > 3$.
- $h'(x) > 0$ for all $x > 4$.

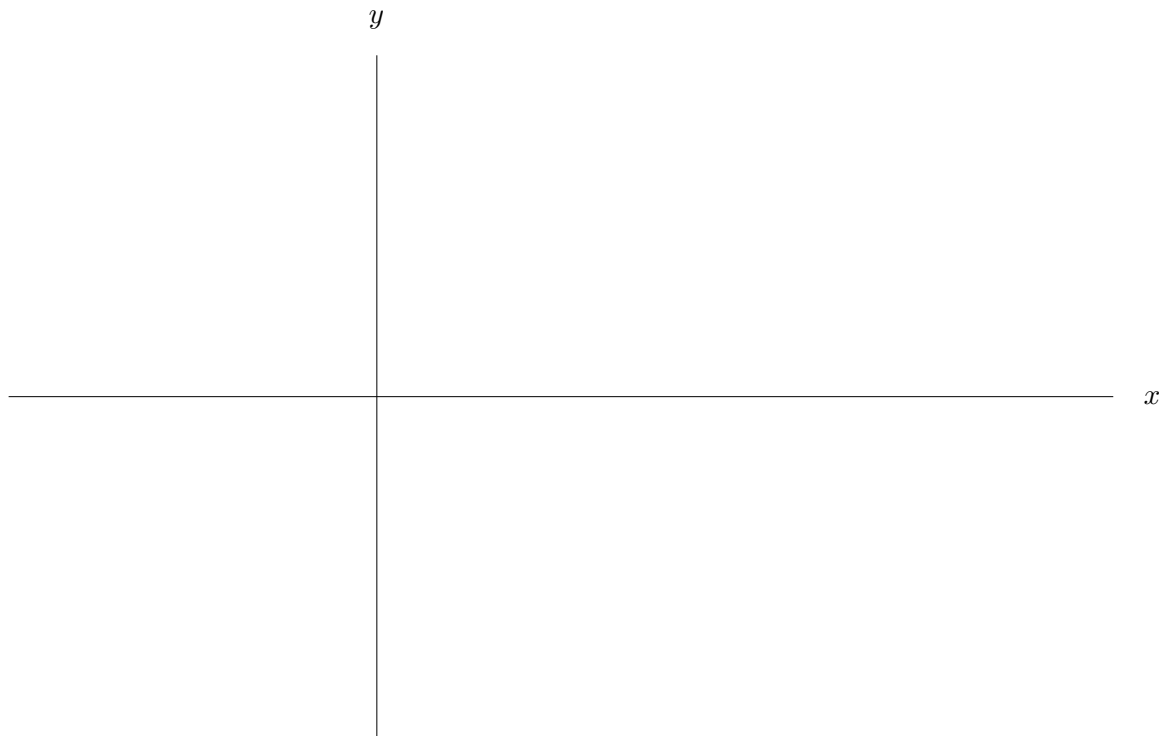
Make sure that your sketch is large and unambiguous.



5. [12 points] A function g defined for all real numbers has the following properties:

- (a) g is differentiable for $-1 \leq x < 4$.
- (b) $g'(x) \leq 0$ for $-1 \leq x < 4$.
- (c) $g''(x) > 0$ for $2 < x < 4$.
- (d) $g(4) = -2$.
- (e) $\lim_{x \rightarrow 4} g(x) = 0$.
- (f) g is continuous at $x = 5$ but not differentiable at $x = 5$.
- (g) $g'(0) = 0$.

On the axes below, draw a possible sketch of $y = g(x)$ on the domain $-1 \leq x \leq 6$, including labels.



4. [9 points] Let $P(v) = \begin{cases} v^2 \sin\left(\frac{1}{v}\right) - v \sin(2) & \text{if } v \neq 0 \\ 0 & \text{if } v = 0. \end{cases}$

a. [5 points]

Use the limit definition of the derivative to write down an explicit expression for $P'(0)$.
Your answer should not include the letter P .

Do not attempt to evaluate or simplify the limit.

$$P'(0) =$$

b. [4 points] Use your answer to (a) to estimate $P'(0)$ to the nearest hundredth.

Be sure to include enough clear graphical or numerical evidence to justify your answer.

Answer: $P'(0) \approx$ _____

5. [11 points] Oren, a Math 115 student, realizes that the more caffeine he consumes, the faster he completes his online homework assignments. Before starting tonight's assignment, he buys a cup of coffee containing a total of 100 milligrams of caffeine.

Let $T(c)$ be the number of minutes it will take Oren to complete tonight's assignment if he consumes c milligrams of caffeine. Suppose that T is continuous and differentiable.

- a. [2 points] Circle the ONE sentence below that is best supported by the statement "the more caffeine he consumes, the faster he completes his online homework assignments."

i. $T'(c) \geq 0$ for every value c in the domain of T .

ii. $T'(c) \leq 0$ for every value c in the domain of T .

iii. $T'(c) = 0$ for every value c in the domain of T .

- b. [1 point] Explain, in the context of this problem, why it is reasonable to assume that $T(c)$ is invertible.

- c. [2 points] Interpret the equation $T^{-1}(100) = 45$ in the context of this problem. Use a complete sentence and include units.

- d. [3 points] Suppose that p and k are constants. In the equation $T'(p) = k$, what are the units on p and k ?

Answer: Units on p are _____

Answer: Units on k are _____

- e. [3 points] Which of the statements below is best supported by the equation $(T^{-1})'(20) = -10$? Circle the ONE best answer.

i. If Oren has consumed 20 milligrams of caffeine, then consuming an additional milligram of caffeine will save him about 10 minutes on tonight's assignment.

ii. The amount of caffeine that will result in Oren finishing his homework in 21 minutes is approximately 10 milligrams greater than the amount of caffeine that Oren will need in order to finish his homework in 20 minutes.

iii. The rate at which Oren is consuming caffeine 20 minutes into his homework assignment is decreasing by 10 milligrams per minute.

iv. In order to complete tonight's assignment in 19 rather than 20 minutes, Oren needs to consume about 10 milligrams of additional caffeine.

v. If Oren consumes 20 milligrams of caffeine, then he will finish tonight's assignment approximately 10 minutes faster than if he consumes no caffeine.

3. [10 points] Elphaba the squirrel has been involved in some questionable activity of late and hence is being very cautious. She has made eye contact with a human standing near her multiple times and is getting anxious that the human is observing her. Let $f(x)$ be Elphaba's anxiety (in "anxious units") after making eye contact with the human for a total of x seconds. Elphaba will panic and run when her anxiety reaches 100 anxious units.

From across the room, the human, Erin, is in fact observing Elphaba while pretending to read a newspaper. The total amount of time Elphaba has spent making eye contact with Erin is a function of the number of times that Erin looks up from the newspaper. Let $g(n)$ be the total amount of time, in seconds, that Erin and Elphaba have spent making eye contact if Erin has looked up from her newspaper n times.

- a. [2 points] Using a complete sentence, give a practical interpretation of the expression $f^{-1}(3) = 10$. Be sure to include units.

- b. [3 points] Below is the first part of a sentence that will give a practical interpretation of the equation

$$f'(25) = 2.$$

Complete the sentence so that the practical interpretation can be understood by someone who knows no calculus. Be sure to include units in your answer.

If Elphaba has already made eye contact with Erin for a total of 25 seconds and she makes eye contact for an additional 0.3 seconds, then

- c. [2 points] Given that $(f^{-1})'(99) = 7$ and $f(62) = 99$, approximate the total length of time Elphaba has to spend making eye contact with Erin before she will panic and run.

- d. [3 points] Which of the following sentences gives a correct interpretation of the quantity $g^{-1}(f^{-1}(50))$? *Circle the ONE best answer.*

- i. When Erin has looked up from her newspaper 50 times, Elphaba's anxiety is at $g^{-1}(f^{-1}(50))$ anxious units.
- ii. When Erin has looked up from her newspaper 50 times, Erin and Elphaba have spent $g^{-1}(f^{-1}(50))$ seconds making eye contact.
- iii. If Erin has looked up from her newspaper $g^{-1}(f^{-1}(50))$ times then Elphaba's anxiety is 50 anxious units.
- iv. If Erin and Elphaba have made eye contact for a total of 50 seconds then Erin has looked up from her newspaper $g^{-1}(f^{-1}(50))$ times.
- v. When Erin and Elphaba have made eye contact for a total of 50 seconds then Elphaba's anxiety is at 50 anxious units.

1. [13 points] For each problem below, circle **ALL** of the statements that **MUST** be true. (The four parts (a)-(d) are independent of each other. No explanations are required.)
- a. [3 points] Suppose f is a differentiable function which is concave up on its entire domain, $(-\infty, \infty)$.
- $\lim_{x \rightarrow 1} f(x) = f(1)$.
 - $f(2) \geq f(1)$.
 - $f'(2) \geq f'(1)$.
- b. [3 points] Suppose that $h(t)$ gives the height of a ball, measured in feet above ground level, t seconds after it is thrown off a bridge. Assume that the derivative of h is given by the formula $h'(t) = -32t + 64$.
- The ball reaches its maximum height 2 seconds after being thrown.
 - The ball reaches a maximum height of 64 feet from the ground.
 - The bridge is 64 feet off the ground.
- c. [4 points] Suppose that A and B are positive constants and $A < B$.
- $(\ln e^A)(\ln e^B) = A + B$
 - $\ln(10^{-A}) < 0$
 - $\ln(A^2 + B) = 2 \ln A + \ln B$
 - $\log A < \log B$
- d. [3 points] Suppose that $f(x) = -Ae^{-Bx}$ for some positive constants A and B .
- $f'(x) > 0$ for all x
 - f' is increasing
 - f is increasing

5. [12 points] A paperback book (definitely not a valuable calculus textbook, of course) is dropped from the top of Dennison hall (which is 40 m high) towards a very large, upward pointing fan. The average velocity of the book between time $t = 0$ and later times is shown in the table of data below (in which t is in seconds and the velocities are in m/s).

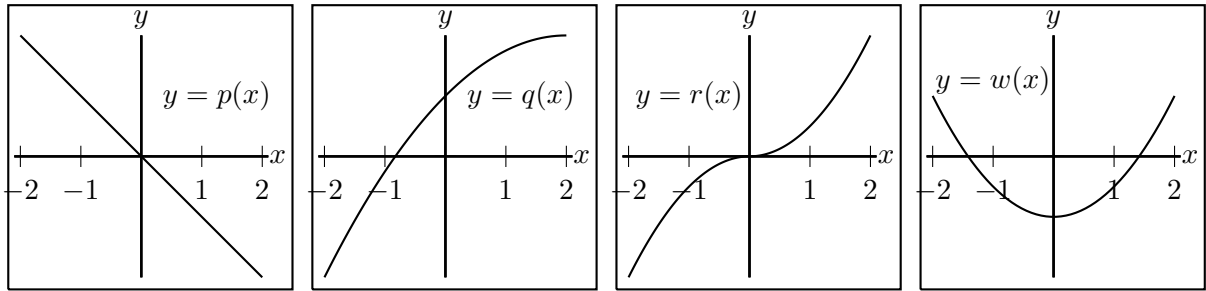
between $t = 0$ seconds and $t =$	1	2	3	4	5
average velocity is	-5	-10	-11.67	-9	-7.2

- a. [8 points] Fill in the following table of values for the height $h(t)$ of the book (measured in meters). Show how you obtain your values.

t	0	1	2	3	4	5
$h(t)$	40	_____	_____	_____	_____	_____

- b. [4 points] Based on your work from (a), is $h''(1) > 0$, < 0 , or $= 0$? Is $h''(3) > 0$, < 0 , or $= 0$? Explain.

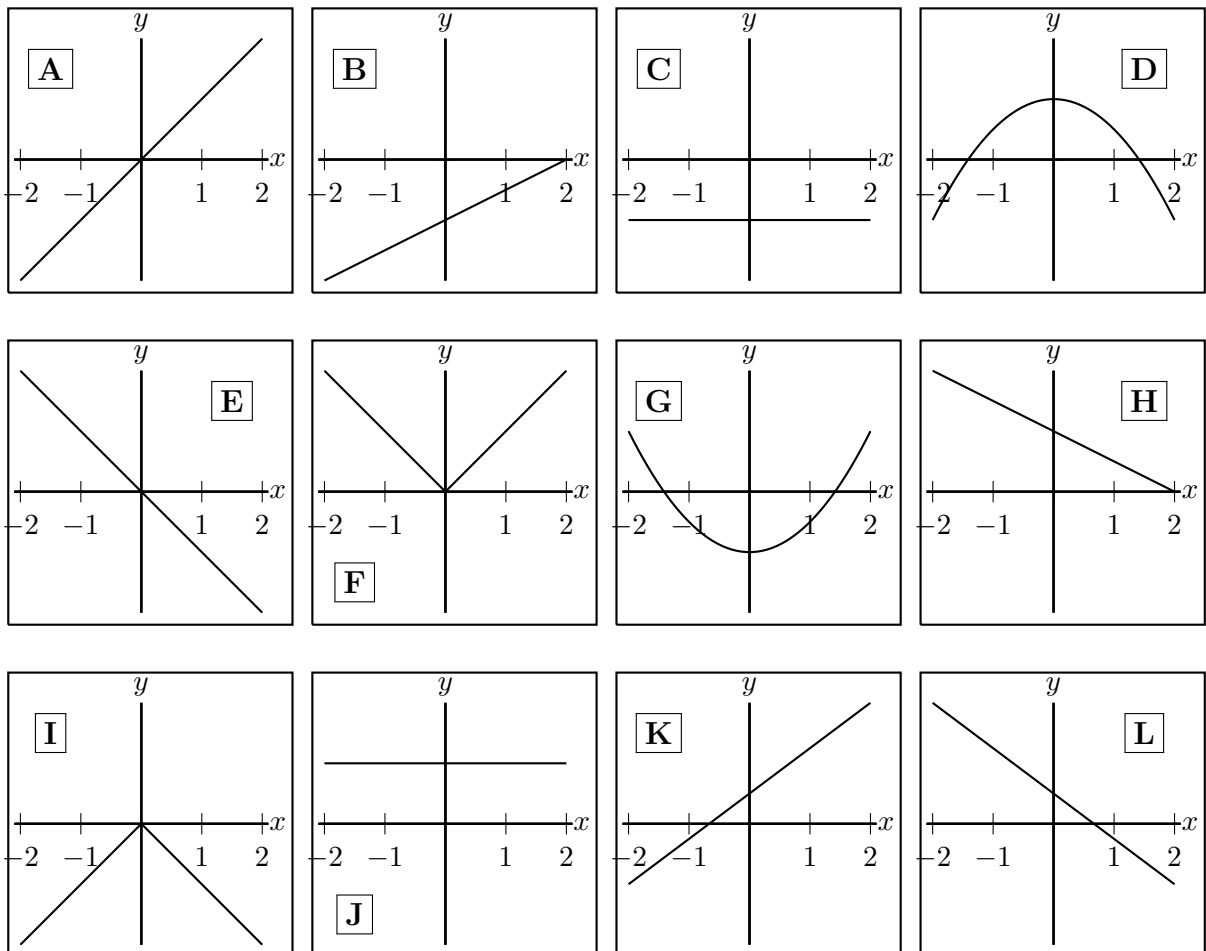
10. [8 points] The graphs of four differentiable functions p , q , r , and w are shown below.



Graph of p' : _____ Graph of q' : _____ Graph of r' : _____ Graph of w' : _____

For each function above, choose the ONE graph from the choices A-L below that best indicates the behavior of the derivative of that function. Write the capital letter of your choice on the provided answer blank. You may use a letter more than once if necessary. Any unclear answers will be marked as incorrect. (Note that the scale on the y -axis is not indicated on any of the graphs and may vary between the graphs.)

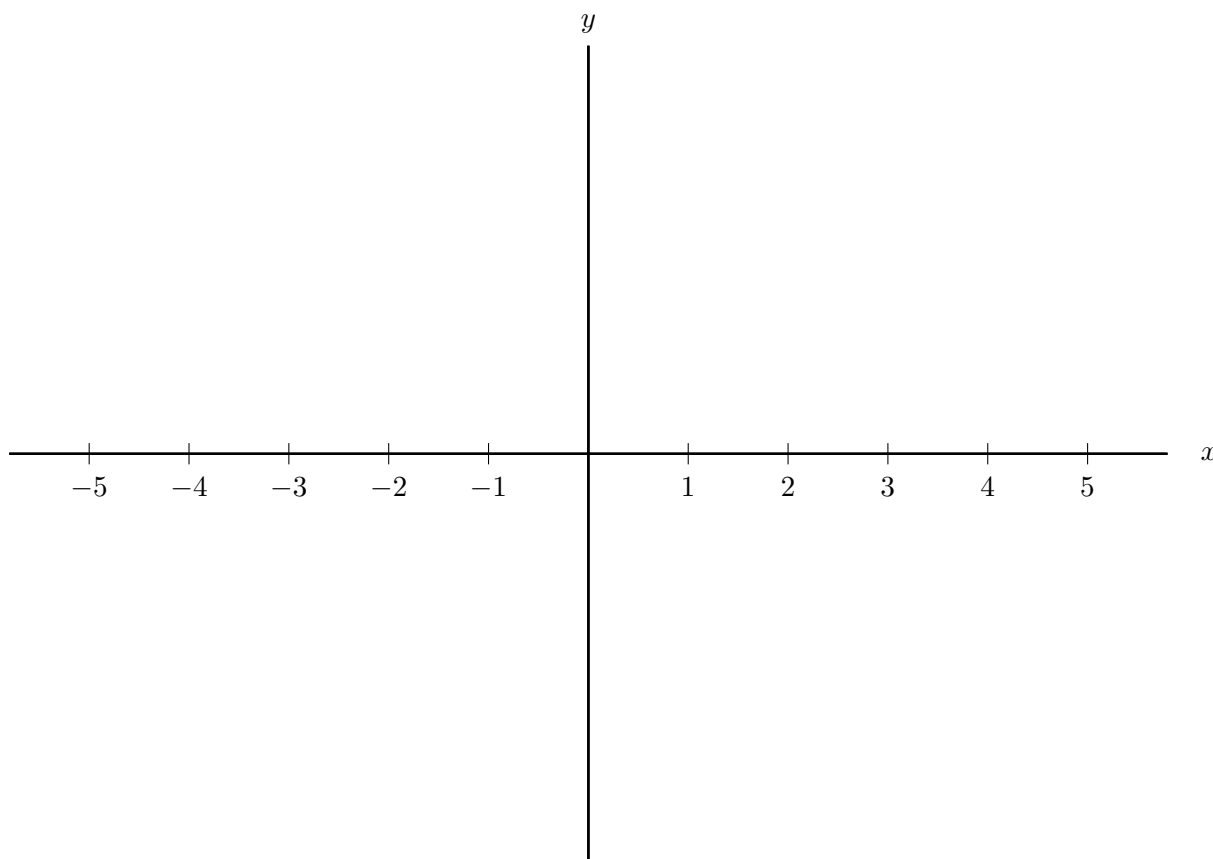
Answer Choices:



8. [8 points] On the axes provided below, sketch the graph of a single function $y = g(x)$ satisfying all of the following:

- $g(x)$ is defined for all x in the interval $-5 < x < 5$.
- $g'(x) > 0$ for all $x < 0$.
- $g(x)$ has a point of discontinuity at $x = 1$.
- The average rate of change of $g(x)$ between $x = -2$ and $x = 2$ is 0.
- $g(x) > 0$ for all $x > 3$.
- $g'(x) < 0$ for all $x > 4$.

Make sure that your sketch is large and unambiguous.



9. [3 points] Find all vertical and horizontal asymptotes of the graph of

$$g(x) = \frac{k(x-a)(x-b)}{(x-a)(x-c)^2}$$

where a , b , c , and k are constants with $a < b < c < k$. If there are none, write NONE.

Horizontal asymptote(s): _____

Vertical asymptote(s): _____

7. [10 points] Sebastian has chartered a helicopter which is rising straight up in the air, but he is scared of heights. Let $A(w)$ be Sebastian's fear (in "scared units") when he is w km above the ground. For $0 < w \leq 2$, a formula for $A(w)$ is given by

$$A(w) = \frac{w^2 + 2}{w^w + 1}.$$

- a. [5 points] Use the limit definition of the derivative to write an explicit expression for the instantaneous rate of change of Sebastian's fear, in scared units per km, when he is 1.5 km above the ground. *Your answer should not involve the letter A. Do not attempt to evaluate or simplify the limit.*

Answer: $A'(1.5) =$

- b. [5 points] When he has reached a height of 2 km above the ground Sebastian gets control of his fear and his fear starts decreasing at a constant rate of 0.8 scared units per km. Write a formula for a piecewise-defined continuous function $A(w)$ giving Sebastian's fear, in scared units, for $0 < w < 3$.

Answer: $A(w) = \left\{ \begin{array}{l} \end{array} \right.$

1. [10 points] Laquita decides to visit an amusement park during Fall Break and rides several roller coasters, including the Classic Amazing Looping Coaster and the Ultra Mountain. Let $R(t)$ be the distance, in feet, that the CAL Coaster has moved along the track t seconds after the ride begins. The ride lasts a total of 60 seconds. Several values of $R(t)$ are shown in the following table.

t	0	10	25	30	40	45	55	60
$R(t)$	0	496	1103	1327	1817	2136	2718	3141

For parts a.– c., remember to show your work and reasoning clearly.

- a. [2 points] Find the average velocity of the CAL Coaster during the last 15 seconds of the ride, i.e. for $45 \leq t \leq 60$. *Include units.*

Answer: _____

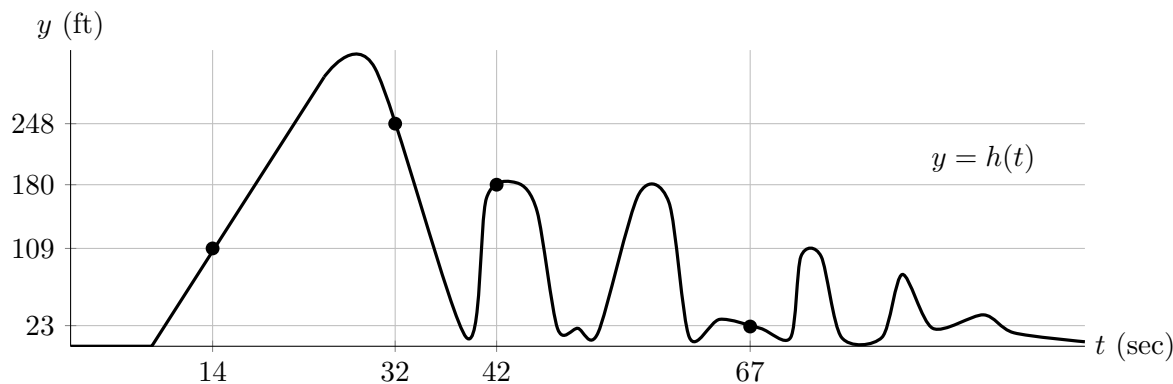
- b. [2 points] Estimate the instantaneous velocity of the CAL Coaster 30 seconds after the ride begins. *Include units.*

Answer: _____

- c. [2 points] Estimate $R'(55)$.

Answer: $R'(55) \approx$ _____

- d. [4 points] Let $h(t)$ be Laquita's height, in feet, above the ground, t seconds after her ride on the Ultra Mountain begins. A graph of $h(t)$ is shown below.



Let the quantities I–V be defined as follows:

- I. The number 0.
- II. Laquita's instantaneous vertical velocity, in ft/sec, at $t = 14$.
- III. $h'(32)$
- IV. Laquita's average vertical velocity, in ft/sec, between $t = 14$ and $t = 42$.
- V. Laquita's instantaneous vertical velocity, in ft/sec, at $t = 67$.

Rank the quantities in order from least to greatest by filling in the blanks below with the options I–V. You do not need to show your work.

_____ < _____ < _____ < _____ < _____