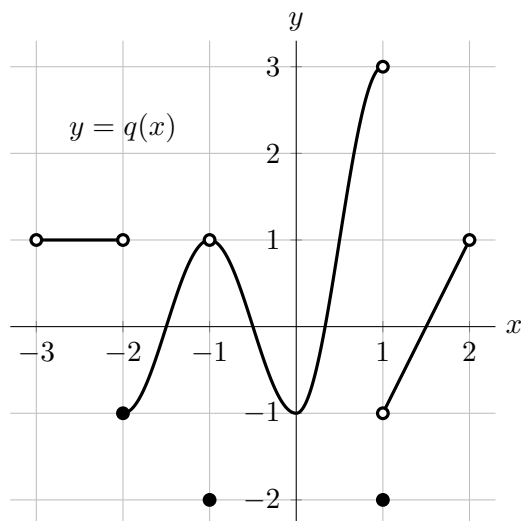


3. [10 points] The entire graph of a function q is shown below. Note that $q(x)$ is linear on the interval $1 < x < 2$.



Throughout this problem, you do not need to explain your reasoning.

For each of parts **a.**– **c.** below, circle all of the listed values satisfying the given statement. If there are no such values, circle NONE.

- a. [2 points] For which of the following values of a does $\lim_{t \rightarrow a} q(t)$ exist?

$a = -2$ $a = -1$ $a = 0$ $a = 1$ NONE

- b. [2 points] For which of the following values of b is $q(x)$ continuous at $x = b$?

$b = -2$ $b = -1$ $b = 0$ $b = 1$ NONE

- c. [2 points] For which of the following values of c is $\lim_{x \rightarrow c^+} q(x) = q(c)$?

$c = -2$ $c = -1$ $c = 0$ $c = 1$ NONE

For each of parts **d.** and **e.** below, if the limit does not exist (including the case of limits that diverge to ∞ or $-\infty$), write DNE.

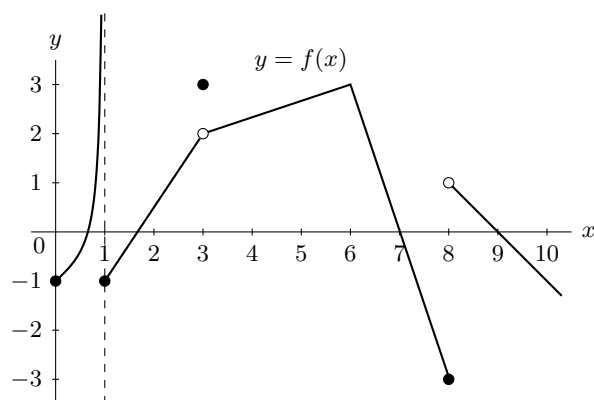
- d. [2 points] Evaluate the following expression: $\lim_{k \rightarrow 0} \frac{q(1.21 + k) - q(1.21)}{k}$.

Answer: _____

- e. [2 points] Evaluate the following expression: $\lim_{s \rightarrow -1} q(q(s))$.

Answer: _____

8. [12 points] A portion of the graph of a function f is shown below.



- a. [2 points] Give all values c in the interval $0 < c < 10$ for which $\lim_{x \rightarrow c} f(x)$ does not exist. If there are none, write NONE.

Answer: $c =$ _____

- b. [2 points] Give all values c in the interval $0 < c < 10$ for which $\lim_{x \rightarrow c^+} f(x)$ does not exist. If there are none, write NONE.

Answer: $c =$ _____

- c. [2 points] Give all values c in the interval $0 < c < 10$ for which $f(x)$ is not continuous at c . If there are none, write NONE.

Answer: $c =$ _____

- d. [6 points] With f as shown in the graph above, define a function g by the formula

$$g(x) = \begin{cases} \frac{B + 2x^2 + 3x^3 + Ax^5}{12 + 6x^3 + 4x^5} & \text{if } x \leq 0 \\ f(x) & \text{if } 0 < x < 10 \end{cases}$$

where A and B are nonzero constants.

Find values of A and B so that both of the following conditions hold.

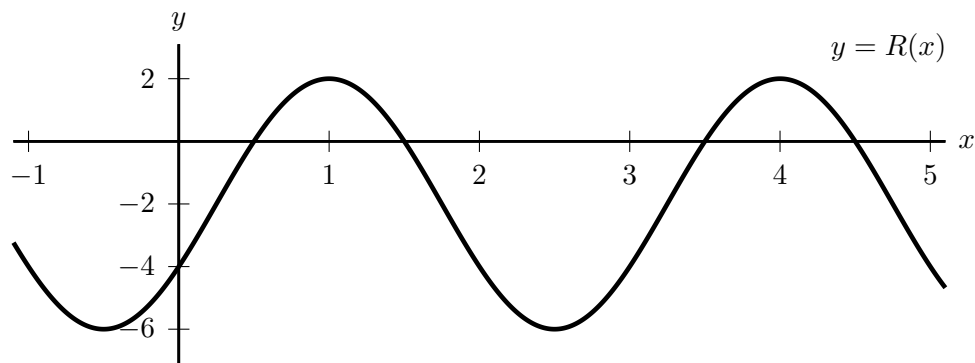
- $g(x)$ is continuous at $x = 0$.
- $\lim_{x \rightarrow -\infty} g(x) = \frac{1}{2}$.

If no such values exist, write NONE in the answer blanks.

Be sure to show your work or explain your reasoning.

Answer: $A =$ _____ and $B =$ _____

8. [6 points] Given below is the graph of a sinusoidal function $R(x)$.



Find a possible formula for $R(x)$.

Answer: $R(x) =$ _____

9. [4 points] The table below gives several values of a function $w(x)$.

x	4.5	4.9	4.99	5	5.01	5.1	5.5
$w(x)$	-0.879	-0.154	-0.015	0	0.060	0.630	3.750

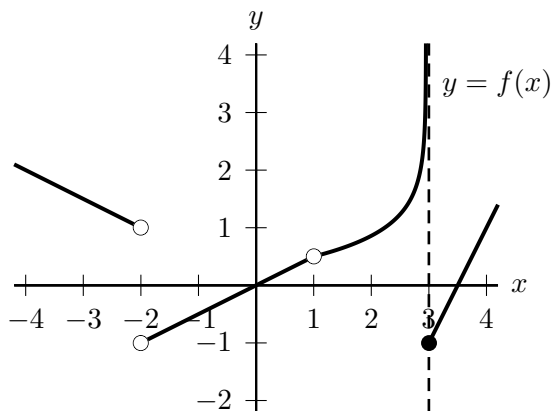
Use the information in the table above to estimate the following limit.

$$\lim_{h \rightarrow 0^-} \frac{w(5+h)}{h}$$

Clearly show any computations that you use to make this estimate.

Answer: $\lim_{h \rightarrow 0^-} \frac{w(5+h)}{h} \approx$ _____

6. [11 points] Below is the graph of a portion of a function $f(x)$.



a. [2 points] Give all values of a in the interval $-4 < a < 4$ that are not in the domain of $f(x)$. If there are none, write NONE.

Answer: _____

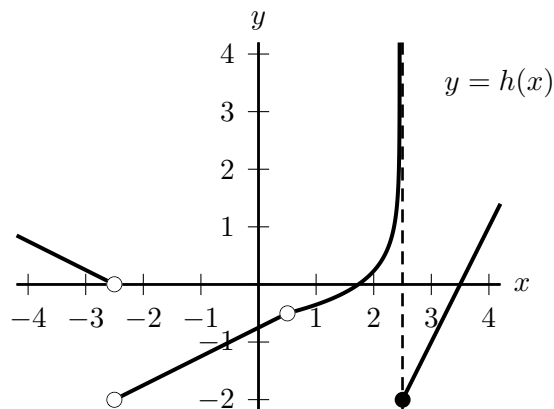
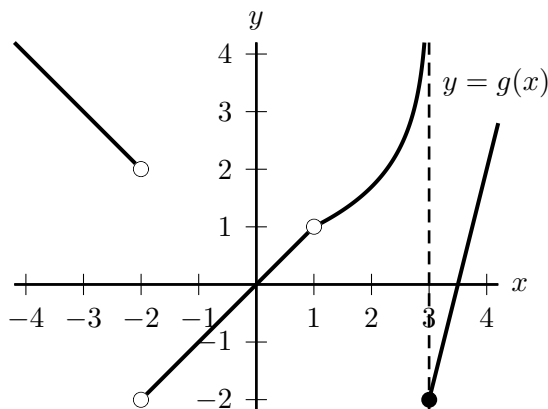
b. [2 points] Give all values of a in the interval $-4 < a < 4$ where $f(x)$ is not continuous at $x = a$. If there are none, write NONE.

Answer: _____

c. [2 points] Give all values of a in the interval $-4 < a < 4$ where $\lim_{x \rightarrow a} f(x)$ does not exist. If there are none, write NONE.

Answer: _____

d. [5 points] The graphs below show portions of two other functions $g(x)$ and $h(x)$ which are transformations of $f(x)$. Express $g(x)$ and $h(x)$ as transformations of $f(x)$.



Answer: $g(x) =$ _____ and $h(x) =$ _____

4. [10 points] For each of the following, give a *formula* for a single function satisfying all of the listed properties. If there is no function satisfying all the properties, circle NO SUCH FUNCTION EXISTS.

Note: If “NO SUCH FUNCTION EXISTS” is circled, then any formula you have written will not be graded.

- a. [3 points] A *polynomial* $p(t)$ with the following three properties:
- The degree of $p(t)$ is three.
 - $p(t) \rightarrow -\infty$ as $t \rightarrow \infty$.
 - $p(0) = -4$.

Answer: $p(t) =$ _____ OR Circle: NO SUCH FUNCTION EXISTS

- b. [3 points] An *exponential function* $q(v)$ with the following three properties:
- $q(1) = 3$.
 - $\lim_{v \rightarrow 0} q(v) = 12$.
 - $\lim_{v \rightarrow \infty} q(v) = 0$.

Answer: $q(v) =$ _____ OR Circle: NO SUCH FUNCTION EXISTS

- c. [4 points] A *rational function* $r(x)$ with the following three properties:
- The line $x = 2$ is a vertical asymptote of the graph of $y = r(x)$.
 - The line $y = -3$ is a horizontal asymptote of the graph of $y = r(x)$.
 - $r(5) = 0$.

Answer: $r(x) =$ _____ OR Circle: NO SUCH FUNCTION EXISTS

2. [12 points] A scientist is growing a very large quantity of mold. Initially, the mass of mold grows exponentially, but after many hours, the mass stabilizes at 24 kilograms. Suppose that t hours after the scientist begins, the mass of mold, in kilograms, can be modeled by the function M defined by the equation

$$M(t) = \begin{cases} 0.41e^{0.72t} & \text{if } 0 \leq t \leq 5 \\ \frac{2t^3}{at^b + c} & \text{if } t > 5. \end{cases}$$

- a. [4 points] Find the value of k between 0 and 5 so that $M(k) = 1$. Then interpret the equation $M(k) = 1$ in the context of this problem. Use a complete sentence and include units.

Answer: $k =$ _____

Interpretation:

- b. [8 points] Assuming that M is a continuous function of t , determine $\lim_{t \rightarrow \infty} M(t)$, and find the values of a , b , and c .

Answers: $\lim_{t \rightarrow \infty} M(t) =$ _____ $a =$ _____

$b =$ _____ $c =$ _____

10. [9 points] Suppose data is collected at a U-M basketball game held at Crisler Center. Let $E(t)$ be the total amount of electricity, in megawatt-hours (MWh), that has been used by Crisler Center during the first t minutes of the basketball game, which starts at exactly 7:00 pm. Assume that E is invertible and that both E and E^{-1} are differentiable.

- a. [3 points] Suppose b and c are positive constants. Use a complete sentence to give a practical interpretation of the equation

$$E(30 + b) = E(30) + c$$

in the context of this problem. Your sentence should involve the constants b and c but not “ E ”. Be sure to include units.

- b. [3 points] Fill in the two answer blanks below to write a single mathematical equality involving the derivative of either E or E^{-1} which supports the following claim:

“During the basketball game, Crisler Center uses about 1.8 MWh of electricity during the first 3 seconds after 7:45 pm.”

Answer: _____ = _____

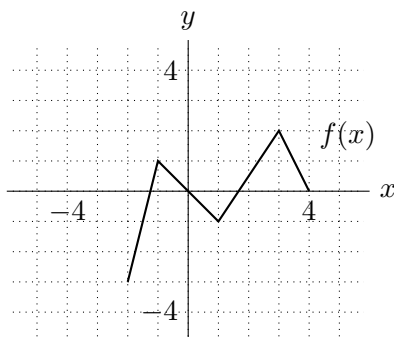
- c. [3 points] Which of the sentences below best expresses the meaning of the equation

$$E^{-1}(20) = 1.5E^{-1}(12)$$

in the context of this problem? (Circle the one best choice.)

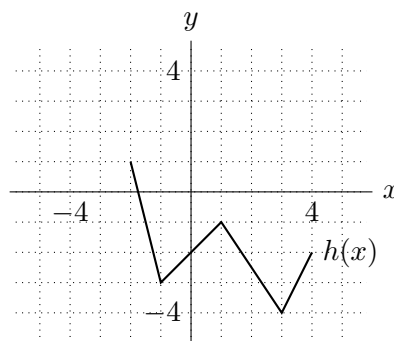
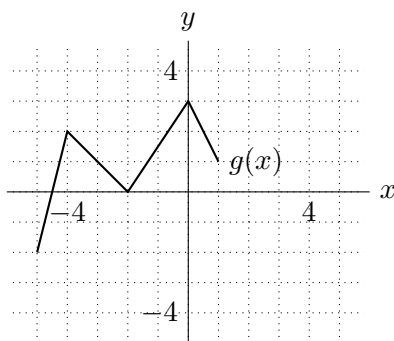
- A. Crisler Center uses 50% more electricity during the first 20 minutes after the game starts than during the first 12 minutes after the game starts.
- B. It takes half as long for Crisler Center to use the first 12 MWh of electricity during the game than for it to use the next 8 MWh.
- C. Crisler Center uses 50% as much electricity during the first 20 minutes after the game starts than during the first 12 minutes after the game starts.
- D. It takes 50% longer for Crisler Center to have used a total of 20 MWh of electricity during the game than for it to use the first 12 MWh.
- E. Crisler Center uses twice as much electricity during the first 20 minutes after the game starts than during the next 12 minutes.
- F. It takes 50% less time for Crisler Center to have used a total of 12 MWh of electricity during the game than for it to use the first 20 MWh.

7. [15 points] The graph of a function $f(x)$ is shown below. The domain of $f(x)$ is $-2 \leq x \leq 4$.



You do not need to show work on this page.

a. [6 points] Each of the functions $g(x)$ and $h(x)$ shown below is a transformation of the function $f(x)$. Write a formula for each function in terms of $f(x)$.



$g(x) =$ _____ $h(x) =$ _____

b. [4 points] Determine the domain and range of the function $j(x) = -2f(x - 6) + 3$.

Domain: _____ $\leq x \leq$ _____ Range: _____ $\leq y \leq$ _____

c. [5 points] On the axes below, draw a graph of the derivative of $f(x)$.

