Math 115 — Practice for Exam 2

Generated October 12, 2017

Name: SOLUTIONS

Instructor: ______________________________ Section Number: __________

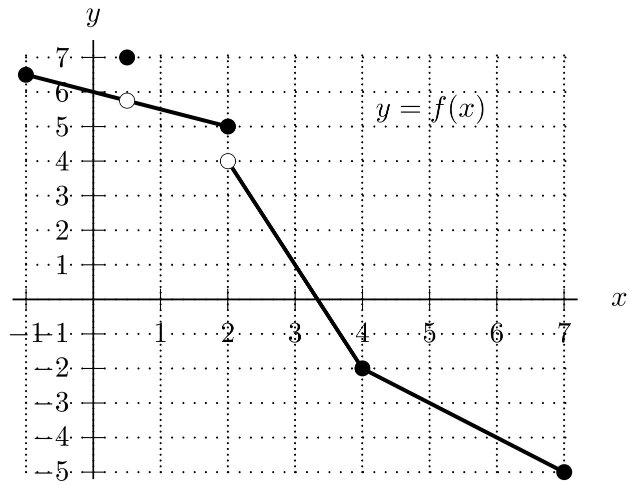
1. This exam has 6 questions. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
2. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you hand in the exam.
3. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
4. Show an appropriate amount of work (including appropriate explanation) for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
5. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
6. If you use graphs or tables to obtain an answer, be certain to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
7. You must use the methods learned in this course to solve all problems.

Semester	Exam	Problem	Name	Points	Score
Fall 2014	2	2		11	
Winter 2013	2	8		12	
Winter 2016	2	2		12	
Winter 2011	2	2		16	
Winter 2012	2	6		16	
Winter 2015	2	1		11	
Total				78	

Recommended time (based on points): 70 minutes

2. [11 points]

Shown to the right is the graph of a function $f(x)$.



Note that you are not required to show your work on this problem. However, limited partial credit may be awarded based on work shown.

Find each of the following values. If the value does not exist, write DOES NOT EXIST.

a. [3 points] Let $h(x) = f(3x+1)$. Find $h'(1)$.

Solution: Since the graph $y = h(x)$ corresponds to the graph of $y = f(x)$ shifted left 1 unit and then horizontally compressed by a factor of $1/3$, $h(x)$ has a "sharp corner" at $x = 1$ so is not differentiable there.

Answer: $h'(1) =$ DOES NOT EXIST

b. [3 points] Let $k(x) = e^{f'(x)}$. Find $k'(6)$.

Solution: By the chain rule, $k'(x) = e^{f'(x)} f''(x)$. So $k'(6) = e^{f'(6)} f''(6) = e^{-1}(0) = 0$.

Answer: $k'(6) =$ 0

c. [2 points] Find $(f^{-1})'(0)$.

Solution: By the formula for the derivative of an inverse,

$$(f^{-1})'(0) = \frac{1}{f'(f^{-1}(0))} = \frac{1}{f'(10/3)} = \frac{1}{-3}.$$

Answer: $(f^{-1})'(0) =$ $-1/3$

d. [3 points] Let $j(x) = \dfrac{f(2x+1)}{x+1}$. Find $j'(1)$.

Solution: Applying the quotient and chain rules, we find that

$$j'(x) = \frac{2f'(2x+1)(x+1) - f(2x+1)(1)}{(x+1)^2}.$$

Thus,

$$j'(1) = \frac{2f'(3)(2) - f(3)}{2^2} = \frac{(2)(-3)(2) - (1)}{4} = \frac{-13}{4}.$$

Answer: $j'(1) =$ $-13/4$

8. [12 points] In the following table, both f and g are differentiable functions of x. In addition, $g(x)$ is an invertible function. Write your answers in the blanks provided. You do not need to show your work.

x	2	3	4	5
$f(x)$	7	6	2	9
$f'(x)$	-2	1	3	2
$g(x)$	1	4	7	11
$g'(x)$	1	2	3	2

a. [3 points] If $h(x) = \frac{g(x)}{f(x)}$, find $h'(4)$.

$h'(4) =$ **-15/4**

b. [3 points] If $k(x) = f(x)g(x)$, find $k'(2)$.

$k'(2) =$ **5**

c. [3 points] If $m(x) = g^{-1}(x)$, find $m'(4)$.

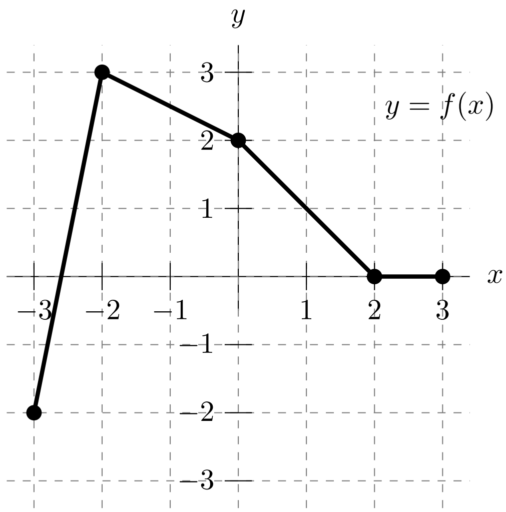
$m'(4) =$ **1/2**

d. [3 points] If $n(x) = f(g(x))$, find $n'(3)$.

$n'(3) =$ **6**

2. [12 points]

Let f be the piecewise linear function with graph shown below.



The table below gives several values of a differentiable function g and its derivative g'.
Assume that both $g(x)$ and $g'(x)$ are invertible.

x	-2	-1	0	2	5
$g(x)$	21	11	5	-1	-3
$g'(x)$	-12	-8	-4	-2	-0.4

You are not required to show your work on this problem. However, limited partial credit may be awarded based on work shown.

For each of parts **a.-f.** below, find the value of the given quantity. If there is not enough information provided to find the value, write "NOT ENOUGH INFO". If the value does not exist, write "DOES NOT EXIST".

a. [2 points] Let $j(x) = e^{g(x)}$. Find $j'(2)$.

Answer: $-\dfrac{2}{e} \approx -0.736$

b. [2 points] Let $k(x) = f(x)f(x+2)$. Find $k'(-1)$.

Answer: -3

c. [2 points] Let $h(x) = 3f(x) + g(x)$. Find $h'(-2)$.

Answer: **DOES NOT EXIST**

d. [2 points] Find $(g^{-1})'(2)$.

Answer: **NOT ENOUGH INFO**

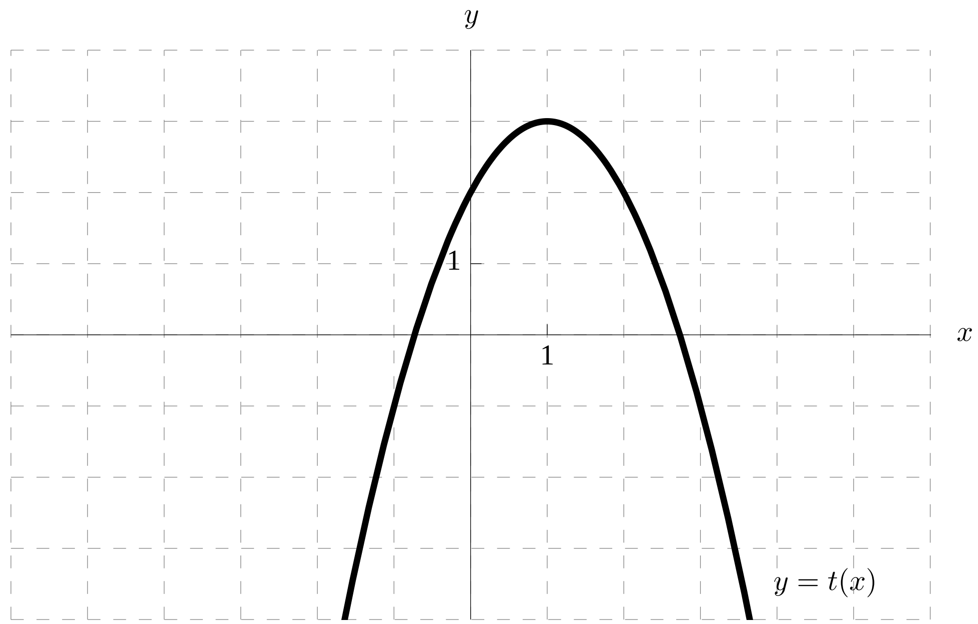
e. [2 points] Let $m(x) = g(f(g(x)))$. Find $m'(2)$.

Answer: **NOT ENOUGH INFO**

f. [2 points] Let $\ell(x) = \dfrac{f(x)}{g(2x)}$. Find $\ell'(-1)$.

Answer: $\dfrac{21(-.5) - 2(2.5)(-12)}{21^2} \approx 0.1122$

2. [16 points]
Graphed below is a function $t(x)$. Define $p(x) = x^2 t(x)$, $q(x) = t(\sin(x))$, $r(x) = \frac{t(x)}{3x+1}$, and $s(x) = t(t(x))$. For this problem, do not assume $t(x)$ is quadratic.



Carefully estimate the following quantities.

a. [4 points] $p'(-1)$

Solution: By the product rule, $p'(x) = 2xt(x) + x^2 t'(x)$. Estimating using the graph, we have

$$p'(-1) = 2(-1)t(-1) + (-1)^2 t'(-1) = (-2)(-1) + 4 = 6.$$

b. [4 points] $q'(0)$

Solution: By the chain rule, $q'(x) = t'(\sin x)\cos x$. Estimating using the graph, we have

$$q'(0) = t'(\sin 0)\cos 0 = t'(0) = 2.$$

c. [4 points] $r'(3)$

Solution: By the quotient rule, $r'(x) = \frac{(3x+1)t'(x) - 3t(x)}{(3x+1)^2}$. Estimating using the graph, we have

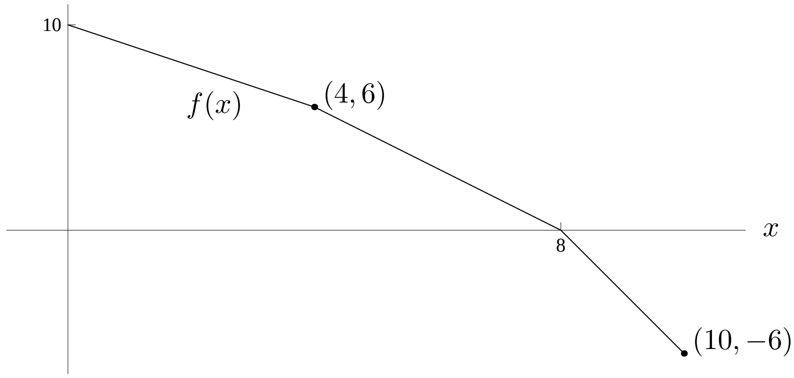
$$r'(3) = \frac{(3(3)+1)t'(3) - 3t(3)}{(3(3)+1)^2} = \frac{-40 - 3(-1)}{100} = -\frac{37}{100}$$

d. [4 points] $s'(0)$

Solution: By the chain rule, $s'(x) = t'(t(x))t'(x)$. Estimating using the graph, we have

$$s'(0) = t'(t(0))t'(0) = t'(2) \cdot 2 = (-2)(2) = -4.$$

6. [16 points] Consider the piecewise linear function $f(x)$ graphed below:



For each function $g(x)$, find the value of $g'(3)$:

a. [4 points] $g(x) = \sin\left([f(x)]^3\right)$

Solution:

$$g'(x) = \cos(f(x)^3) \cdot 3f(x)^2 \cdot f'(x)$$
$$g'(3) = \cos(7^3) \cdot 3 \cdot 7^2 \cdot (-1) = 124.0442.$$

b. [4 points] $g(x) = \dfrac{f(x^2)}{x}$

Solution:

$$g'(x) = \frac{x \cdot f'(x^2) \cdot 2x - f(x^2)}{x^2}$$
$$g'(3) = \frac{3(-3)6 - (-3)}{9} = -5.667.$$

c. [4 points] $g(x) = \ln(f(x)) + f(2)$

Solution:

$$g'(x) = \frac{1}{f(x)} f'(x) + 0$$
$$g'(3) = \frac{1}{7} \cdot (-1) = \frac{-1}{7}.$$

d. [4 points] $g(x) = f^{-1}(x)$

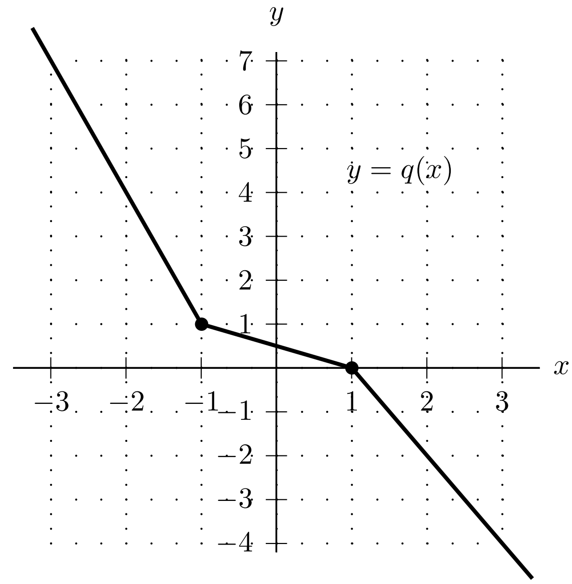
Solution:

$$g'(x) = \frac{1}{f'(f^{-1}(x))}$$
$$g'(3) = \frac{1}{f'(6)} = -\frac{2}{3}.$$

1. [11 points]
Shown to the right is the graph of an invertible piecewise linear function $q(x)$. Note that the graph passes through the points $(-3, 7)$, $(-1, 1)$, $(1, 0)$, and $(3, -4)$.

You are not required to show your work on this problem. However, limited partial credit may be awarded based on work shown.

Find the exact value of each of the quantities below. If there is not enough information provided to find the value, write "NOT ENOUGH INFO". If the value does not exist, write "DOES NOT EXIST".



a. [2 points] Let $r(x) = q^{-1}(x)$. Find $r'(2)$.

Solution: $r'(x) = \dfrac{1}{q'(q^{-1}(x))}$ so $r'(2) = \dfrac{1}{q'(q^{-1}(2))} = \dfrac{1}{-3} = -\dfrac{1}{3}$.

Answer: $r'(2) =$ $-\dfrac{1}{3}$

b. [3 points] Let $w(x) = \dfrac{x}{q(x+1)}$. Find $w'(-2)$.

Solution: By the quotient and chain rules, $w'(x) = \frac{q(x+1) - xq'(x+1)}{(q(x+1))^2}$ (where these quantities are defined). q' is not differentiable at $x = -1$, so $q'(x+1)$ is not defined at $x = -2$. (If $w'(-2)$ were to exist, then since $q(x+1) = \frac{w(x)}{x}$, we would have $q'(-1) = q'(-2+1) = \frac{(-2)w'(-2) - w(-2)}{(-2)^2}$.)

Answer: $w'(-2) =$ DOES NOT EXIST

c. [3 points] Let $v(x) = xq(\sin x)$. Find $v'(\pi)$.

Solution: By the product and chain rules we have $v'(x) = xq'(\sin x)\cos x + q(\sin x)$. So $v'(\pi) = \pi q'(\sin \pi)\cos \pi + q(\sin \pi) = \pi q'(0)(-1) + q(0) = \pi(-1/2)(-1) + (1/2) = \frac{\pi+1}{2}$.

Answer: $v'(\pi) =$ $\dfrac{\pi + 1}{2}$

d. [3 points] Let $j(x) = \ln(q(2x))$. Find $j'(-1)$.

Solution: By the chain rule, we have

$$j'(x) = \frac{1}{q(2x)} \cdot q'(2x) \cdot 2 = \frac{2q'(2x)}{q(2x)} \quad \text{so} \quad j'(-1) = \frac{2q'(-2)}{q(-2)} = \frac{2(-3)}{4} = -\frac{3}{2}.$$

Answer: $j'(-1) =$ $-\dfrac{3}{2}$